Introduction to Phenomenology with Massive Neutrinos in 2024

Concha Gonzalez-Garcia

(YITP-Stony Brook & ICREA-University of Barcelona)

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OUTLINE

- Historic Introduction to the SM of Massless Neutrinos
- Introducing ν mass: Dirac vs Majorana, Lepton mixing, Flavour Oscillations
- Summary of Flavour Oscillation Observations
- Status of 3ν global description
- Explorations beyond 3ν's: steriles,NSI's,Z's…

Discovery of ν's

- At end of 1800's radioactivity was discovered and three types identified: α, β, γ

  β : an electron comes out of the radioactive nucleus.

- Energy conservation $\Rightarrow$ e^- should have had a fixed energy

$$(A, Z) \to (A, Z+1) + e^- \;\Rightarrow\; E_e = M(A, Z+1) - M(A, Z)$$

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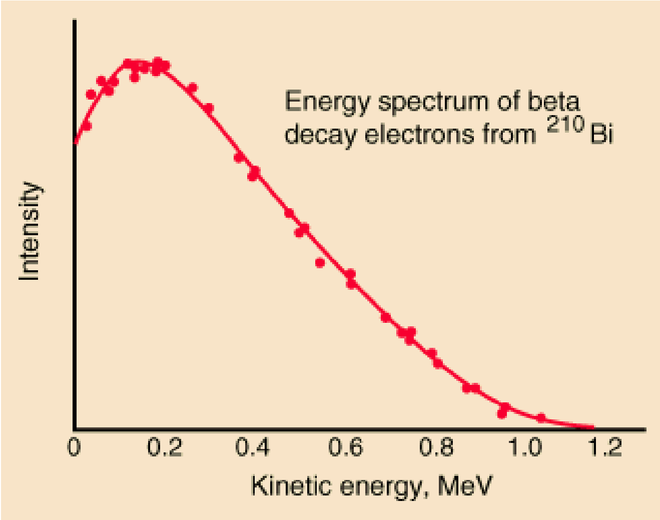

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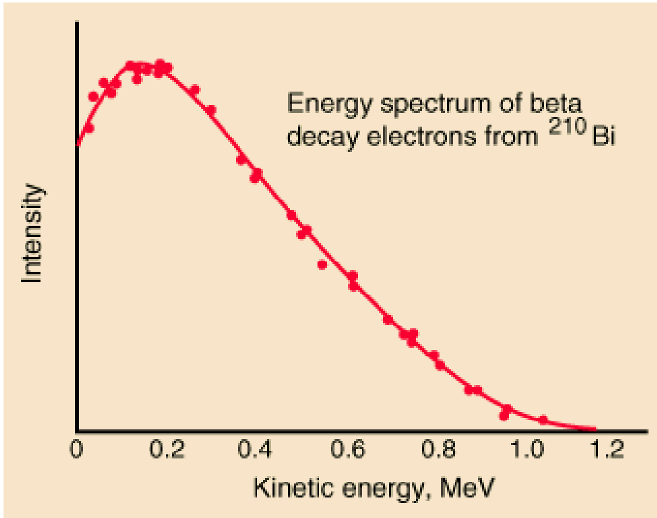

Do we throw away the energy conservation?

Bohr: *we have no argument, either empirical or theoretical, for upholding the energy principle in the case of β ray disintegrations*

Discovery of ν's

• The idea of the neutrino came in 1930, when W. Pauli tried a desperate saving operation of "the energy conservation principle".

In his letter addressed to the *Liebe Radioaktive Damen und Herren* (Dear Radioactive Ladies and Gentlemen), the participants of a meeting in Tubingen. He put forward the hypothesis that a new particle exists as *constituent of nuclei, the neutron* ν, able to explain the continuous spectrum of nuclear beta decay

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$$(A,Z) \to (A,Z+1)+e^- + \nu$$

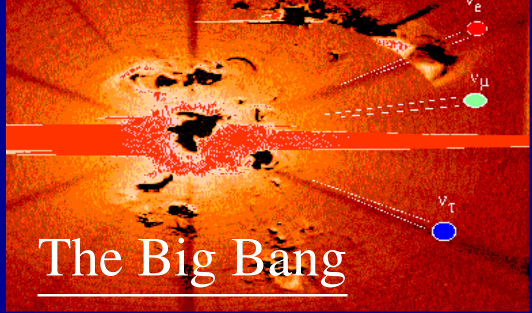
• The ν is light (in Pauli's words:
m_ν *should be of the same order as the* m_e),
neutral and has spin 1/2

Neutrino Detection

Fighting Pauli's "Curse":

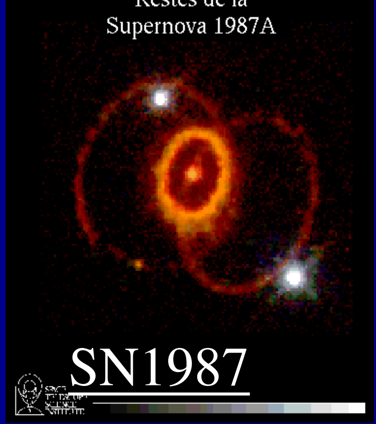
I have done a terrible thing, I have postulated a particle that cannot be detected.

Sources of ν's

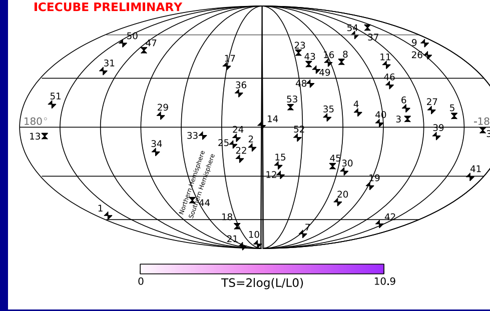


$\rho_\nu = 330/\text{cm}^3$

$p_\nu = 0.0004$ eV



$E_\nu \sim$ MeV



ExtraGalactic

$E_\nu \gtrsim 30$ TeV

The Sun

ν_e

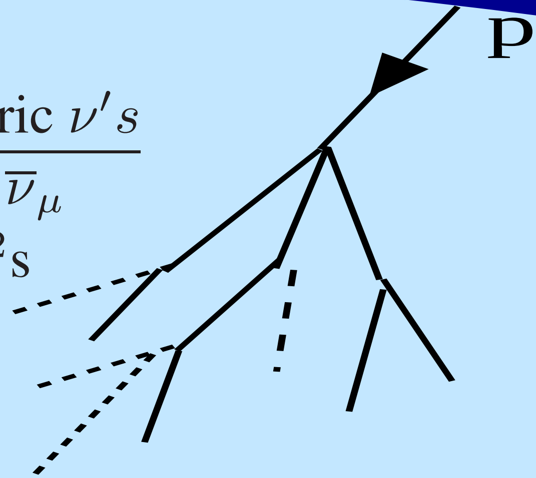
$\Phi_\nu^{Earth} = 6 \times 10^{10} \nu/\text{cm}^2\text{s}$

$E_\nu \sim 0.1$–20 MeV

Atmospheric $\nu's$

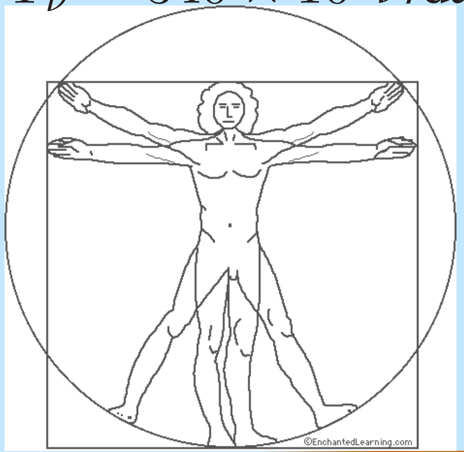
$\nu_e, \nu_\mu, \overline{\nu}_e, \overline{\nu}_\mu$

$\Phi_\nu \sim 1 \nu/\text{cm}^2\text{s}$



Human Body

$\Phi_\nu = 340 \times 10^6 \nu/\text{day}$



Nuclear Reactors

$E_\nu \sim$ few MeV

$\overline{\nu}_e$

Earth's radioactivity

$\Phi_\nu \sim 6 \times 10^6 \nu/\text{cm}^2\text{s}$

Accelerators

$E_\nu \simeq 0.3$–30 GeV

Fermilab — Discovering the Nature of Nature

CERN

KEK

Oak Ridge National Laboratory

NSS

$E_\nu \sim$ MeV

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Neutrino Detection

Problem: Quantitatively: a ν sees a proton of area:

$$\sigma^{\nu p} \sim 10^{-38}\mathrm{cm}^2 \frac{E_\nu}{\mathrm{GeV}}$$

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$$\Phi_\nu^{\mathrm{ATM}} = 1\,\nu/(\,\mathrm{cm}^2\ \mathrm{second})\ \ \mathrm{y}\ \ \ \langle E_\nu\rangle = 1\ \mathrm{GeV}$$

- How many interact?

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$$N_{\text{int}} = \Phi_\nu \times \sigma^{\nu p} \times N_{\text{prot}}^{\text{human}} \times T_{\text{life}}^{\text{human}}$$

$$\left.\begin{array}{l} N_{\text{protons}}^{\text{human}} = \frac{M^{\text{human}}}{gr} \times N_A = 80\text{kg} \times N_A \sim 5 \times 10^{28}\text{protons} \\ T^{\text{human}} = 80\,\text{years} = 2 \times 10^9\ \text{sec} \end{array}\right\}$$

$(M \times T \equiv \text{Exposure})$

$\text{Exposure}_{\text{human}}$

$\sim \text{Ton} \times \text{year}$

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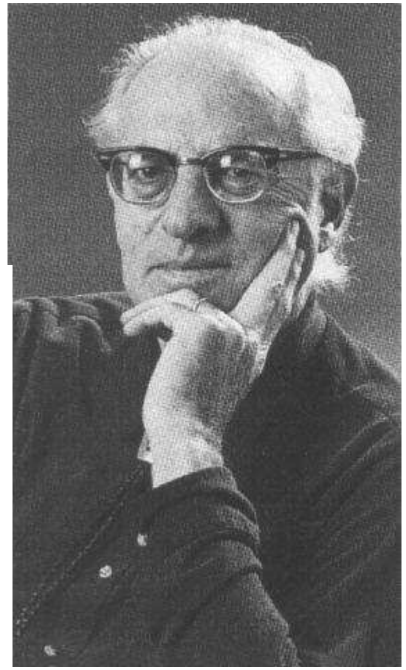
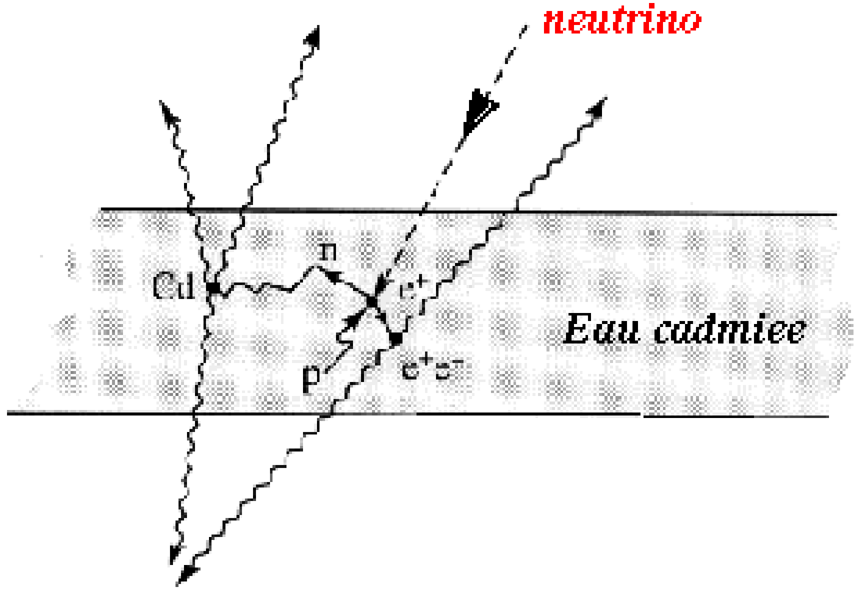
To detect neutrinos we need very intense source and/or
a hugh detector with Exposure $\sim$ KTon $\times$ year

First Neutrino Detection



In 1953 Frederick Reines and Clyde Cowan put a detector near a nuclear reactor (the most intense source available)

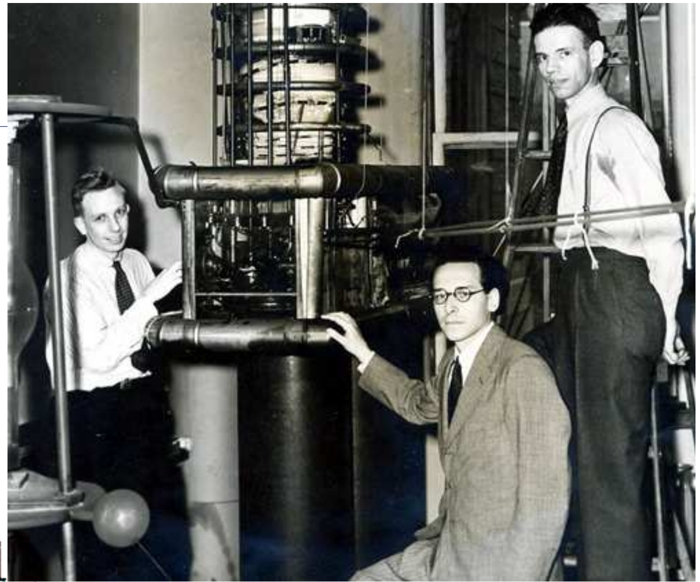
400 l of water
and Cadmium Chloride.



e^+ annihilates with e^- in the water and produces two γ's simultaneouoly.
neutron is captured by por the cadmium and a γ's is emitted 15 msec latter

Reines y Clyde saw clearly this signature: the first neutrino had been detected

Neutrinos = "Left-handed"



Helicity of Neutrinos*

M. GOLDHABER, L. GRODZINS, AND A. W. SUNYAR
Brookhaven National Laboratory, Upton, New York
(Received December 11, 1957)

A COMBINED analysis of circular polarization and resonant scattering of γ rays following orbital electron capture measures the helicity of the neutrino. We have carried out such a measurement with Eu^{152m}, which decays by orbital electron capture. If we assume the most plausible spin-parity assignment for this isomer compatible with its decay scheme,[1] $0-$, we find that the neutrino is "left-handed," i.e., $\boldsymbol{\sigma}_\nu \cdot \hat{p}_\nu = -1$ (negative helicity).

WARNING: Helicity versus Chirality

• We define the chiral projections $\boxed{\mathcal{P}_{R,L} = \frac{1 \pm \gamma_5}{2}}$ $\Rightarrow$ $\psi_L = \frac{1-\gamma_5}{2}\psi$ $\quad$ $\psi_R = \frac{1+\gamma_5}{2}\psi$

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- The Hamiltonian for a massive fermion ψ is $H = \overline{\psi}(x)(-i\sum_j \gamma^j \partial_j + m)\psi(x)$
- 4 states with $(E, \vec{p})$ $(\gamma^\mu p_\mu - m)u_s(\vec{p}) = 0$ $\quad (\gamma^\mu p_\mu + m)v_s(\vec{p}) = 0$ $\quad s = 1, 2$

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- Since $[H, \gamma_5] \neq 0$ and $[\vec{P}, \vec{J}] \neq 0$ $\quad [\ \vec{J} = \vec{L} + \frac{\vec{\Sigma}}{2} \quad (\Sigma^i = -\gamma^0\gamma^5\gamma^i)\ ]$

  $\Rightarrow$ Neither Chirality nor J_i can characterize the fermion simultaneously with $E, \vec{p}$

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$$\mathcal{P}_\pm = \tfrac{1}{2}\left(1 \pm 2\vec{J}\frac{\vec{P}}{|\vec{P}|}\right) = \tfrac{1}{2}\left(1 \pm \vec{\Sigma}\frac{\vec{P}}{|\vec{P}|}\right)$$

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$$\vec{\Sigma}\,\vec{P}\,\psi = -\gamma^0\gamma^5\vec{\gamma}\,\vec{p}\,\psi = -\gamma^0\gamma^5\gamma^0 E\,\psi = \gamma^5 E\psi \Rightarrow \text{For } m = 0\ \mathcal{P}_\pm = \mathcal{P}_{R,L}$$

Only for massless fermions Helicity and chirality states are the same.

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ν in the SM

• The SM is a gauge theory based on the symmetry group

$$SU(3)_C \times SU(2)_L \times U(1)_Y \Rightarrow SU(3)_C \times U(1)_{EM}$$

• 3 Generations of Fermions:

$(1,2,-\frac{1}{2})$	$(3,2,\frac{1}{6})$	$(1,1,-1)$	$(3,1,\frac{2}{3})$	$(3,1,-\frac{1}{3})$
L_L	Q^i_L	E_R	U^i_R	D^i_R
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L$	$\begin{pmatrix}u^i \\ d^i\end{pmatrix}_L$	e_R	u^i_R	d^i_R
$\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L$	$\begin{pmatrix}c^i \\ s^i\end{pmatrix}_L$	μ_R	c^i_R	s^i_R
$\begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$	$\begin{pmatrix}t^i \\ b^i\end{pmatrix}_L$	τ_R	t^i_R	b^i_R

• Spin-0 particle ϕ: $(1,2,\frac{1}{2})$

$$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix} \stackrel{SSB}{\rightarrow} \frac{1}{\sqrt{2}}\begin{pmatrix}0 \\ v+h\end{pmatrix}$$

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$\begin{pmatrix}\nu_e\\ e\end{pmatrix}_L$	$\begin{pmatrix}u^i\\ d^i\end{pmatrix}_L$	e_R	u^i_R	d^i_R
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$$Q_{EM} = T_{L3} + Y$$

- ν's are $T_{L3} = \frac{1}{2}$ components of L_L
- ν's have no strong or EM interactions
- No ν_R ($\equiv$ singlets of gauge group)

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SM Fermion Lagrangian

$$
\begin{aligned}
\mathcal{L} \;=\; & \sum_{k=1}^{3}\sum_{i,j=1}^{3}\overline{Q^i_{L,k}}\gamma^\mu\left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g\frac{\tau_a}{2}\delta_{ij}W^a_\mu - g'\frac{1}{6}\delta_{ij}B_\mu\right)Q^j_{L,k} \\
& +\sum_{k=1}^{3}\sum_{i,j=1}^{3}\overline{U^i_{R,k}}\gamma^\mu\left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{2}{3}\delta_{ij}B_\mu\right)U^j_{R,k} \\
& +\sum_{k=1}^{3}\sum_{i,j=1}^{3}\overline{D^i_{R,k}}\gamma^\mu\left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu + g'\frac{1}{3}\delta_{ij}B_\mu\right)D^j_{R,k} \\
& +\sum_{k=1}^{3}\overline{L_{L,k}}\gamma^\mu\left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu + g'\frac{1}{2}B_\mu\right)L_{L,k} + \overline{E_{R,k}}\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)E_{R,k} \\
& -\sum_{k,k'=1}^{3}\left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi^* U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}
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- Invariant under global rotations

$Q_{L,k} \to e^{i\alpha_B/3}Q_{L,k}$ $\quad U_{R,k} \to e^{i\alpha_B/3}U_{R,k}$ $\quad D_{R,k} \to e^{i\alpha_B/3}D_{R,k}$ $\quad L_{L,k} \to e^{i\alpha_{L_k}}L_{L,k}$ $\quad E_{R,k} \to e^{i\alpha_{L_k}}E_{R,k}$

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& +\sum_{k=1}^{3}\sum_{i,j=1}^{3}\overline{U^i_{R,k}}\gamma^\mu\left(i\partial_\mu - g_s\frac{\lambda_{a,ij}}{2}G^a_\mu - g'\frac{2}{3}\delta_{ij}B_\mu\right)U^j_{R,k} \\
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& +\sum_{k=1}^{3}\overline{L_{L,k}}\gamma^\mu\left(i\partial_\mu - g\frac{\tau_i}{2}W^i_\mu + g'\frac{1}{2}B_\mu\right)L_{L,k} + \overline{E_{R,k}}\gamma^\mu\left(i\partial_\mu + g'B_\mu\right)E_{R,k} \\
& -\sum_{k,k'=1}^{3}\left(\lambda^u_{kk'}\overline{Q}_{L,k}(i\tau_2)\phi^* U_{R,k'} + \lambda^d_{kk'}\overline{Q}_{L,k}\phi D_{R,k'} + \lambda^l_{kk'}\overline{L}_{L,k}\phi E_{R,k'} + h.c.\right)
\end{aligned}$$

- Invariant under global rotations

$Q_{L,k} \to e^{i\alpha_B/3}Q_{L,k}$ $\quad U_{R,k} \to e^{i\alpha_B/3}U_{R,k}$ $\quad D_{R,k} \to e^{i\alpha_B/3}D_{R,k}$ $\quad L_{L,k} \to e^{i\alpha_{L_k}}L_{L,k}$ $\quad E_{R,k} \to e^{i\alpha_{L_k}}E_{R,k}$

$\Rightarrow$ *Accidental ($\equiv$ not imposed)* global symmetry: $U(1)_B \times U(1)_{L_e} \times U(1)_{L_\mu} \times U(1)_{L_\tau}$

$\Rightarrow$ Each lepton flavour, L_i, is conserved

$\Rightarrow$ Total lepton number $L = L_e + L_\mu + L_\tau$ is conserved

SM = massless $\nu's$ and LFC

- A fermion mass can be seen as at a Left-Right transition

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In SM ν's are *Strictly* Massless & Lepton Flavours are *Strictly* Conserved

- We have observed with high (or good) precision:
  - $*$ Atmospheric ν_μ & $\bar{\nu}_\mu$ disappear most likely to ν_τ (**SK**,MINOS, ICECUBE)
  - $*$ Accel. ν_μ & $\bar{\nu}_\mu$ disappear at $L \sim 300/800$ Km (K2K, **T2K, MINOS, NOνA**)
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All this implies that L_α are violated

and There is Physics Beyond SM

Dirac versus Majorana Neutrinos

- In the SM neutral bosons can be of two type:
  - Their own antiparticle such as γ, π^0 ...
  - Different from their antiparticle such as K^0, $\bar{K}^0$...
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⇒ 4 chiral fields

$$\nu_L\ ,\ \nu_R\ ,\ (\nu_L)^C\ ,\ (\nu_R)^C \quad \text{with} \quad \nu = \nu_L + \nu_R \quad \text{and} \quad \nu^C = (\nu_L)^C + (\nu_R)^C$$

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The difference arises when including *a neutrino mass*

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${V_R^\nu}^\dagger M_D V^\nu = \text{diag}(m_1, m_2, m_3)$

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$\Rightarrow$ Total Lepton number is conserved *by construction* (not *accidentally*):

$$\left.\begin{array}{l} U(1)_L \;:\; \nu \to e^{i\alpha}\,\nu \quad \text{and} \quad \overline{\nu} \to e^{-i\alpha}\,\overline{\nu} \\ U(1)_L \;:\; \nu^C \to e^{-i\alpha}\,\nu^C \quad \text{and} \quad \overline{\nu^C} \to e^{i\alpha}\,\overline{\nu^C} \end{array}\right\} \Rightarrow \mathcal{L}_{\text{mass}}^{(\text{Dirac})} \to \mathcal{L}_{\text{mass}}^{(\text{Dirac})}$$

Adding ν Mass: Majorana Mass

• One does not introduce ν_R but uses that the field $(\nu_L)^c$ is right-handed, so that one can write a Lorentz-invariant mass term

$$\mathcal{L}_{\rm mass}^{(\rm Maj)} = -\frac{1}{2}\overline{\nu_L^c} M_M^\nu \nu_L + \text{h.c.} \equiv -\frac{1}{2}\sum_k m_k \overline{\nu}_i^M \nu_i^M$$

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M_M^ν =Majorana mass for ν's is *symmetric* $V^{\nu T} M_M V^\nu = \text{diag}(m_1, m_2, m_3)$

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$\Rightarrow$ Breaks Total Lepton Number $\Rightarrow \mathcal{L}_{\rm mass}^{\rm (Maj)}$ not generated at any order in the SM

ν Mass $\Rightarrow$ Lepton Mixing

- CC and mass for 3 charged leptons ℓ_i and N neutrinos in weak basis $\nu^W \equiv \begin{pmatrix} \nu_{L,e} \\ \nu_{L,\mu} \\ \nu_{L,\tau} \\ (\nu_{R,1})^C \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$

$$\mathcal{L}_{CC}+\mathcal{L}_M = -\frac{g}{\sqrt{2}}\sum_{i=1}^{3}\overline{\ell^W_{L,i}}\gamma^\mu \nu_i^W W_\mu^+ - \sum_{i,j=1}^{3}\overline{\ell^W_{L,i}} M_{\ell\, ij}\ell^W_{R,j} - \frac{1}{2}\sum_{i,j=1}^{N}\overline{\nu_i^{cW}} M_{\nu\, ij}\nu_j^W + \text{h.c.}$$

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$$V^{\ell\,\dagger}_L M_\ell V^\ell_R = \text{diag}(m_e, m_\mu, m_\tau)$$

$V^\ell_{L,R} \equiv$ Unitary 3×3 matrices

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• $U_{\text{LEP}} \equiv 3\times N$ matrix $\; U_{\text{LEP}}U^\dagger_{\text{LEP}} = I_{3\times 3}$ but in general $\; U^\dagger_{\text{LEP}}U_{\text{LEP}} \neq I_{N\times N}$

$$U^{ij}_{\text{LEP}} = \sum_{k=1}^{3} P^\ell_{ii} V_L^{\ell\,\dagger\, ik}\, V^{\nu\, kj} P^\nu_{jj}$$

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- For example for 3 Dirac ν : 3 Mixing angles + 1 Dirac Phase

$$U_{\rm LEP} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{21} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

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- For 3 Majorana ν : 3 Mixing angles + 1 Dirac Phase + 2 Majorana Phases

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Effects of ν Mass: Flavour Transitions

- Flavour ($\equiv$ Interaction) basis (production and detection): ν_e, ν_μ and ν_τ
- Mass basis (free propagation in space-time): ν_1, ν_2 and ν_3 $\ldots$

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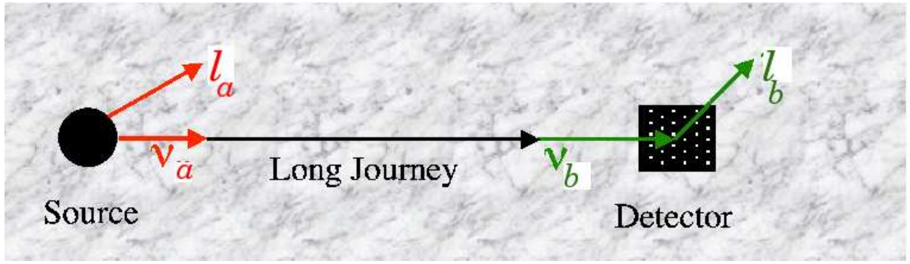
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\mathrm{LEP}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \ldots \end{pmatrix} \Rightarrow$$

Flavour not conserved in propagation



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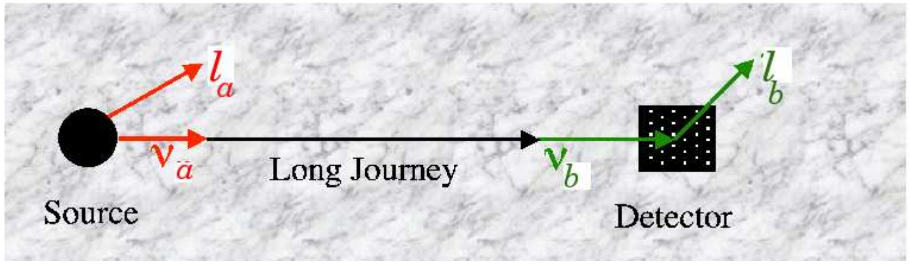

- The probability $P_{\alpha\beta}$ of producing neutrino with flavour α and detecting with flavour β has to depend on:
  - Misalignment between interaction and propagation states ($\equiv U$)
  - Difference between propagation eigenvalues
  - Propagation distance

Mass Induced Flavour Oscillations in Vacuum

- If neutrinos have mass, a weak eigenstate $|\nu_\alpha\rangle$ produced in $l_\alpha + N \rightarrow \nu_\alpha + N'$ is a linear combination of the mass eigenstates ($|\nu_i\rangle$)

$$|\nu_\alpha\rangle = \sum_{i=1}^{n} U^*_{\alpha i} \, |\nu_i\rangle$$

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- it can be detected with flavour β with probability

$$P_{\alpha\beta} = |\langle\nu_\beta|\nu_\alpha(t)\rangle|^2 = |\sum_{j=1}^{n}\sum_{i=1}^{n} U^*_{\alpha i} U_{\beta j} \langle\nu_j|\nu_i(t)\rangle|^2$$

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- We call E_i the neutrino energy and m_i the neutrino mass
- Under the approximations:

  $(\mathbf{1})$ $|\nu\rangle$ is a *plane wave* $\Rightarrow$ $|\nu_i(t)\rangle = \mathrm{e}^{\,-i\,E_i t}|\nu_i(0)\rangle$ and using $\langle \nu_j | \nu_i \rangle = \delta_{ij}$

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with $\Delta_{ij} = (E_i - E_j)t$

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$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

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$$E_i = \sqrt{p_i^2 + m_i^2} \simeq p_i + \frac{m_i^2}{2E_i}$$

  $(\mathbf{3})$ Lowest order in mass $p_i \simeq p_j = p \simeq E$

$$\frac{\Delta_{ij}}{2} = \frac{(m_i^2 - m_j^2)\,L}{4\,E} = 1.27\frac{m_i^2 - m_j^2}{\mathrm{eV}^2}\,\frac{L/E}{\mathrm{Km/GeV}}$$

Mass Induced Flavour Oscillations in Vacuum

- The oscillation probability:

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$\to$ conserves CP

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– The last piece $2\sum_{i<j}\mathrm{Im}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]\sin\left(\Delta_{ij}\right)$ opposite sign for $\overline{\nu}$

$\to$ violates CP

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$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i<j}^{n} \mathrm{Re}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right) + 2\sum_{i<j}\mathrm{Im}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]\sin\left(\Delta_{ij}\right)$$

$$\frac{\Delta_{ij}}{2} = \frac{(E_i - E_j)L}{2} = 1.27\frac{(m_i^2 - m_j^2)}{\mathrm{eV}^2}\frac{L/E}{\mathrm{Km/GeV}}$$

– The first term $\delta_{\alpha\beta} - 4\sum_{i<j}^{n} \mathrm{Re}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]\sin^2\left(\frac{\Delta_{ij}}{2}\right)$ equal for $\overline{\nu}$ $(U \to U^*)$

$\to$ conserves CP

– The last piece $2\sum_{i<j}\mathrm{Im}[U_{\alpha i}U^*_{\beta i}U^*_{\alpha j}U_{\beta j}]\sin\left(\Delta_{ij}\right)$ opposite sign for $\overline{\nu}$

$\to$ violates CP

– If $\alpha = \beta \Rightarrow$ $\mathrm{Im}[U_{\alpha i}U^*_{\alpha i}U^*_{\alpha j}U_{\alpha j}] = \mathrm{Im}[|U^\star_{\alpha i}|^2|U_{\alpha j}|^2] = 0$

$\Rightarrow$ CP violation observable only for $\beta \neq \alpha$

Mass Induced Flavour Oscillations in Vacuum

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  - $U_{\alpha j}$ The mixing angles (and Dirac phases)

and on Two *set-up* Parameters:
  - E The neutrino energy
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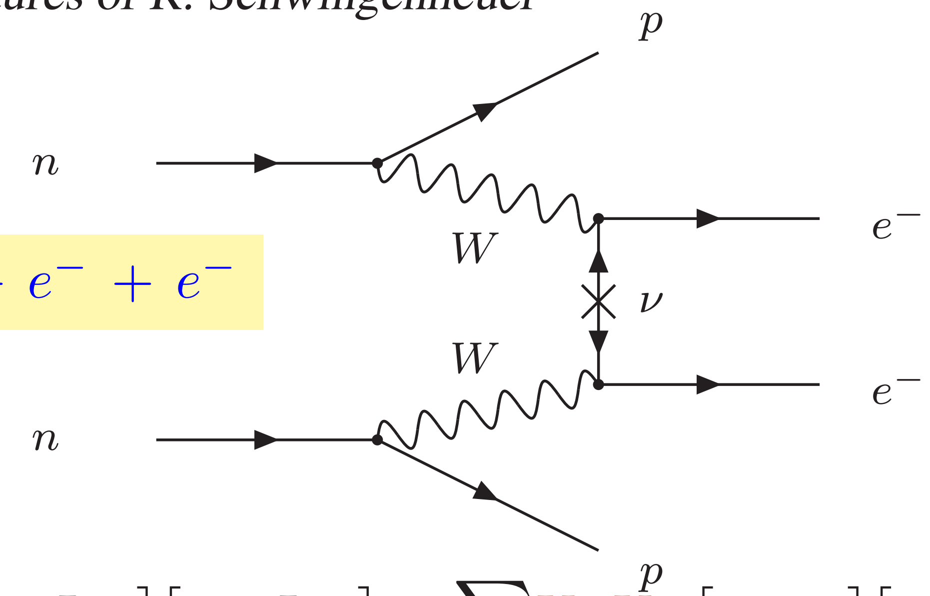
Dirac or Majorana? ν-less Double-β Decay

Lectures of R. Schwingenheuer

$$(A,Z) \to (A,Z+2) + e^- + e^-$$



- Amplitude includes $\left[\overline{e}\gamma^\mu L\nu_e\right]\left[\overline{e}\gamma^\mu L\nu_e\right] = \sum_{ij} U_{ei}U_{ej}\left[\overline{e}\gamma^\mu\nu_i\right]\left[\overline{e}\gamma^\mu\nu_j\right]$

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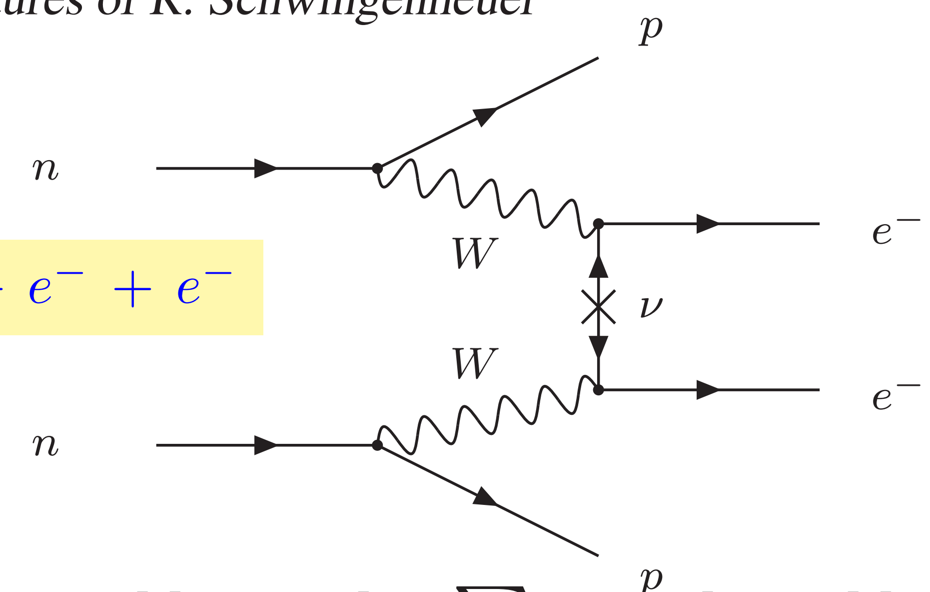

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    $\Rightarrow$ no same state $\Rightarrow$ Amplitude = 0
  - If ν_i Majorana $\Rightarrow$ $\nu_i = \nu_i^c$ *annihilates and creates* a *neutrino=antineutrino*

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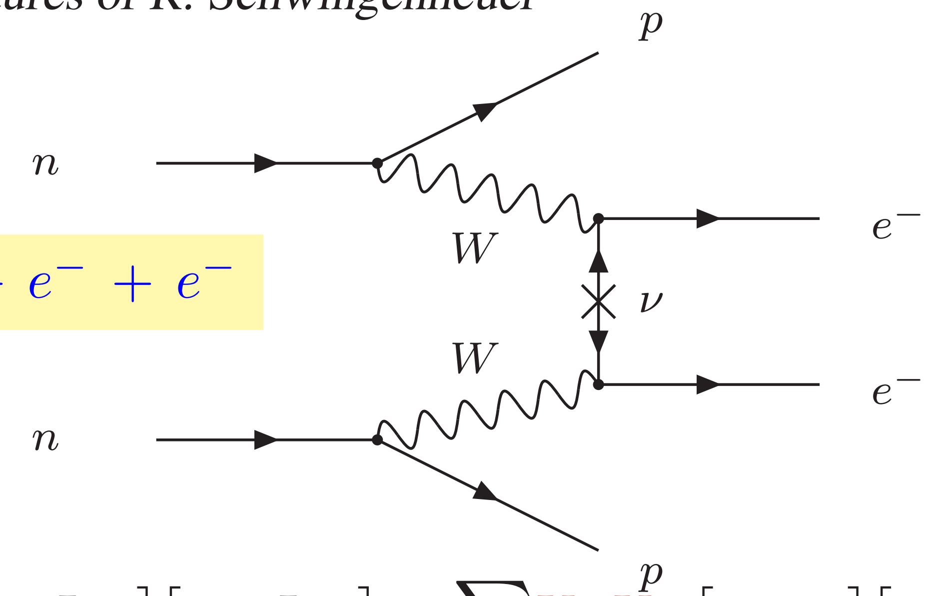

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- If Majorana m_ν only source of L-violation

$\Rightarrow$ Amplitude of ν-less-$\beta\beta$ decay is proportional to $\langle m_{ee}\rangle = \sum_j U_{ej}^2 m_j$

Neutrino Mass Scale: Tritium β Decay

- Fermi proposed a kinematic search of ν_e mass from beta spectra in ^3H beta decay

$$^3\text{H} \rightarrow ^3\text{He} + e + \overline{\nu}_e$$

- For "allowed" nuclear transitions, the electron spectrum is given by phase space alone

$$K(T) \equiv \sqrt{\frac{dN}{dT}\frac{1}{C p\, E\, F(E)}} \propto \sqrt{(Q-T)\sqrt{(Q-T)^2 - m^2_{\nu_e}}}$$

$T = E_e - m_e$, Q= maximum kinetic energy, (for ^3H beta decay $Q = 18.6$ KeV)

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$$m_\nu = 0 \;\Rightarrow\; T_{\rm max} = Q$$
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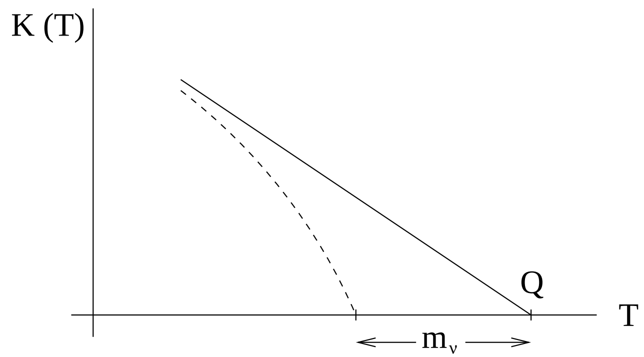

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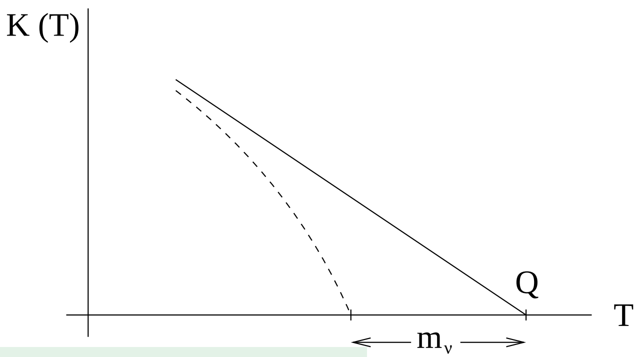
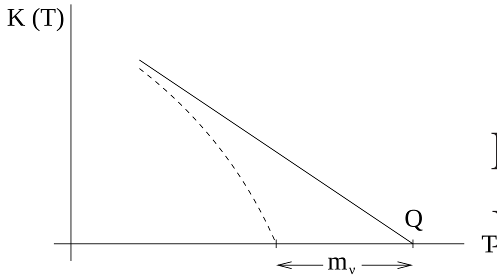


– At present only a bound: $m^{\text{eff}}_{\nu_e} < 0.8$ eV (at 90 % CL) (Katrin)

– Katrin operating can improve present sensitivity to $m^{\text{eff}}_{\nu_e} \sim 0.3\,\text{eV}$

Probes of Mass Scale

Single β decay : Pure kinematics, Dirac or Majorana ν's, only model independent

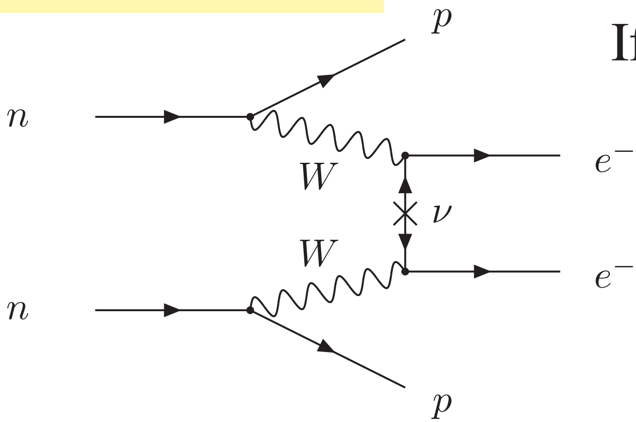


$$m^2_{\nu_e} = \sum m^2_j |U_{ej}|^2$$

Present bound: $m_{\nu_e} \leq 0.8$ eV (90% CL KATRIN 2021)

Katrin (20XX) Sensitivity to $m_{\nu_e} \sim 0.2\,\text{eV}$

ν-less Double-β decay: $\Leftrightarrow$ Majorana $\nu's$



If m_ν only source of ΔL $\quad T^{0\nu}_{1/2} = \frac{m_e}{G_{0\nu}\, M^2_{\text{nucl}}\, m^2_{ee}}$

$$m_{ee} = |\sum U^2_{ej} m_j|$$

Present Bounds: $m_{ee} < 0.04\text{–}0.2$ eV

COSMO for Dirac or Majorana

$$\sum m_i$$

Lecture S. Pastor

Back up Slides

The Other Flavours

ν coming out of a nuclear reactor is $\overline{\nu}_e$ because it is emitted together with an e^-

Question: Is it different from the muon type neutrino ν_μ that could be associated to the muon? Or is this difference a theoretical arbitrary convention?

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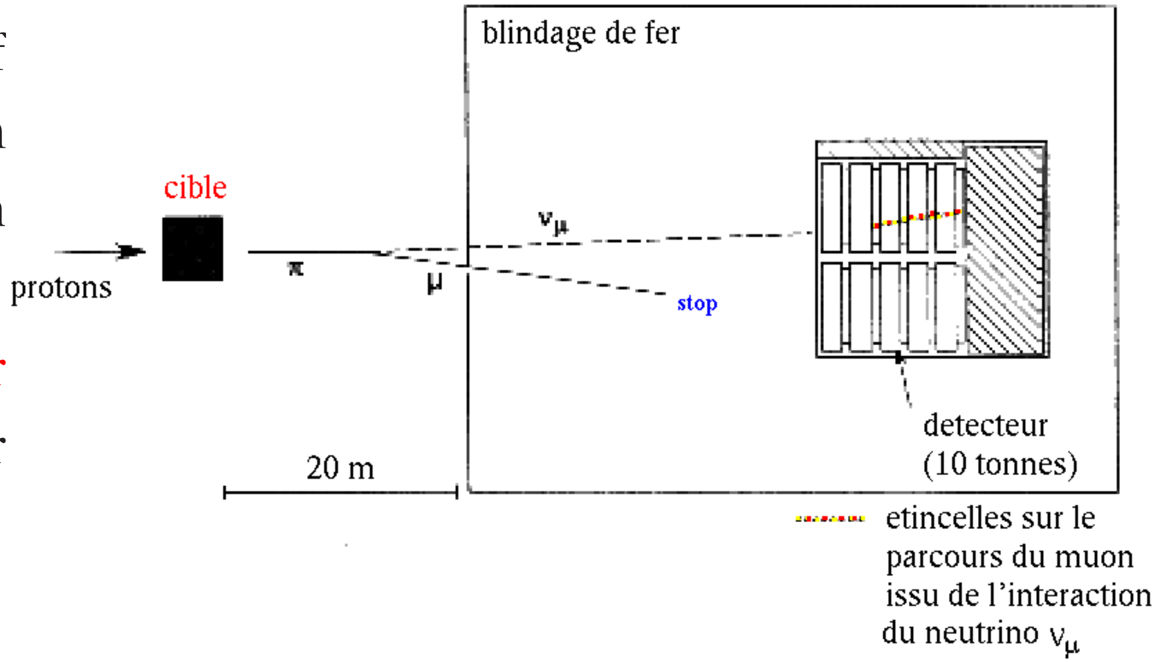

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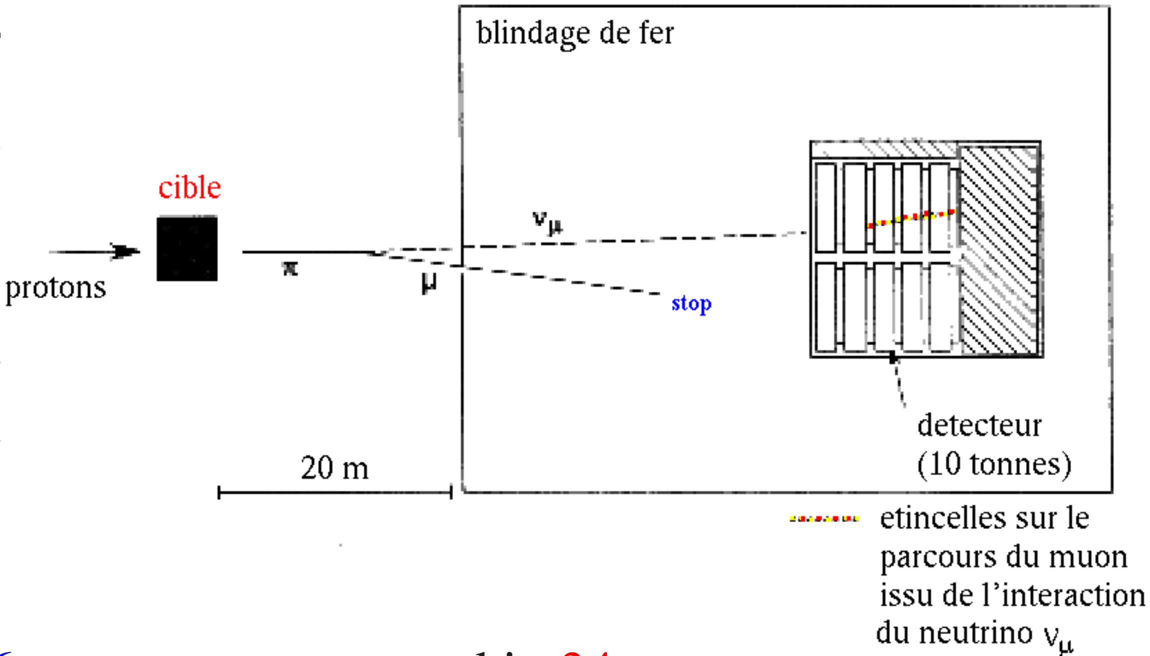

They observe 40 ν interactions: in 6 an e^- comes out and in 34 a μ^- comes out.

If $\nu_\mu \equiv \nu_e \Rightarrow$ equal numbers of μ^- and $e-$

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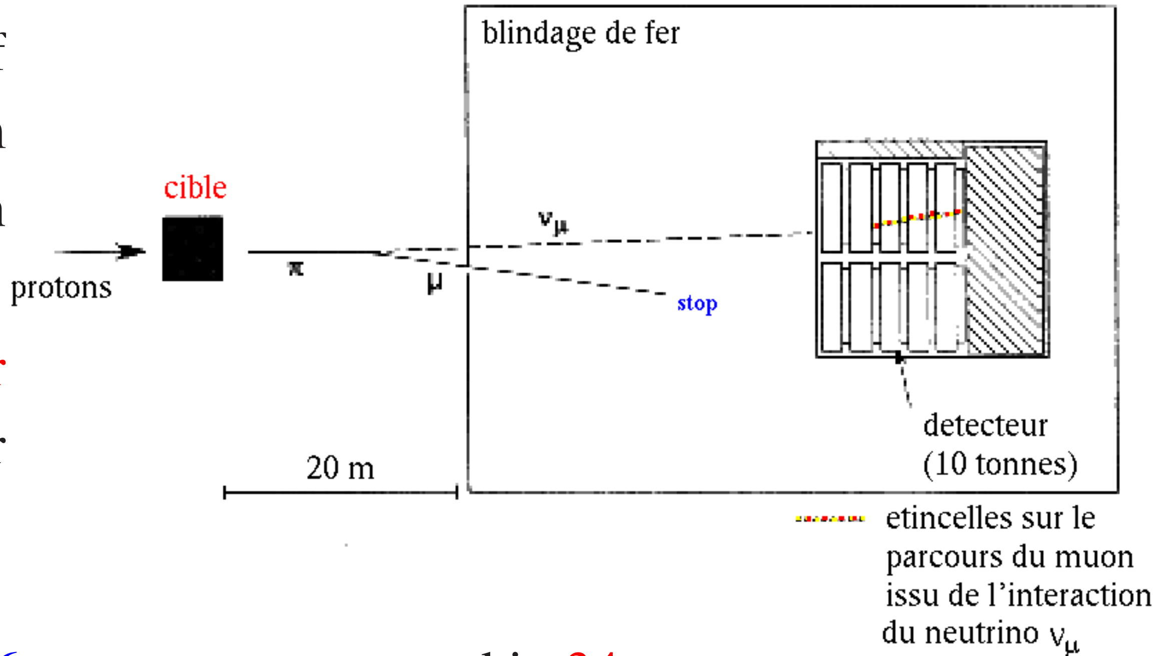

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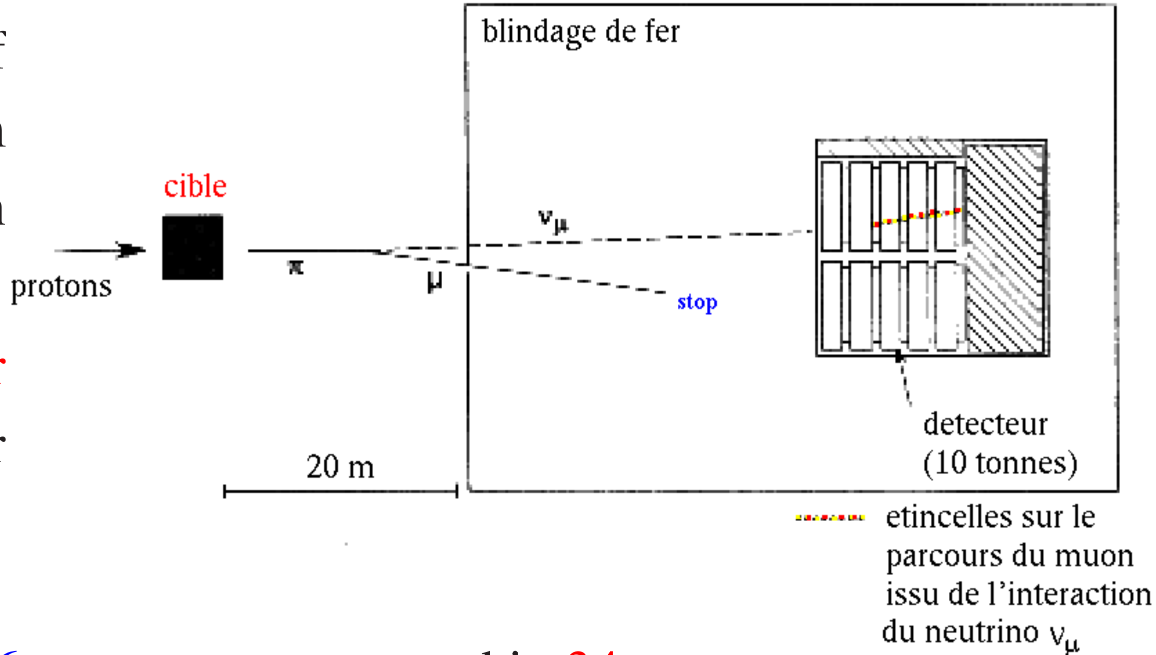

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In 1977 Martin Perl discovers the particle tau $\equiv$ the third lepton family.

The ν_τ was observed by DONUT experiment at FNAL in 1998 (*officially* in Dec. 2000).

Neutrino Helicity

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- The neutrino helicity was measured in 1957 in a experiment by Goldhaber et al.

- Using the electron capture reaction

$$e^- + {}^{152}Eu \to \nu + {}^{152}Sm^*$$
$$\to {}^{152}Sm + \gamma$$

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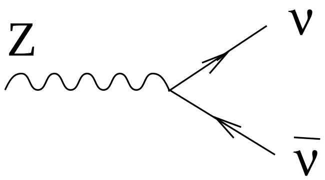
- Goldhaber et al found γ had negative helicity $\Rightarrow$ ν has helicity -1

Number of Neutrinos

• The counting of light left-handed neutrinos is based on the family structure of the SM assuming a universal diagonal NC coupling:



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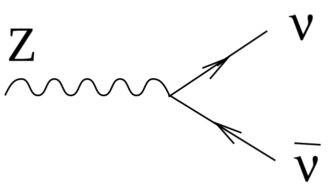


$$j_Z^\mu = \sum_\alpha \bar{\nu}_{\alpha L}\gamma^\mu \nu_{\alpha L}$$

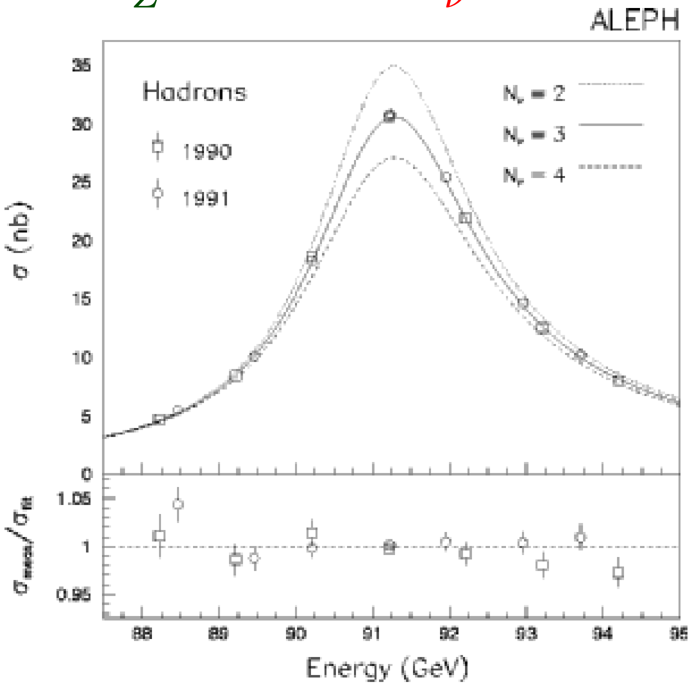
• For $m_{\nu_i} < m_Z/2$ one can use the total Z-width Γ_Z to extract N_ν

$$N_\nu = \frac{\Gamma_{\rm inv}}{\Gamma_\nu} \equiv \frac{1}{\Gamma_\nu}(\Gamma_Z - \Gamma_h - 3\Gamma_\ell) = \frac{\Gamma_\ell}{\Gamma_\nu}\left[\sqrt{\frac{12\pi R_{h\ell}}{\sigma_h^0 m_Z^2}} - R_{h\ell} - 3\right]$$

Γ_{inv}= the invisible width

Γ_h= the total hadronic width

Γ_l= width to charged lepton



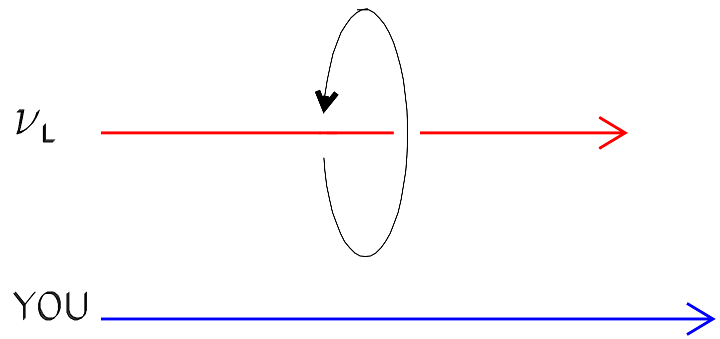
Leads $N_\nu = 2.9840 \pm 0.0082$

Neutrinos always "Left-Handed" $\equiv$ Massless

- If ν had a mass they would not go to the speed of light:

  $\Rightarrow$ the direction of its momentum depends on the reference frame

  So in one reference frame



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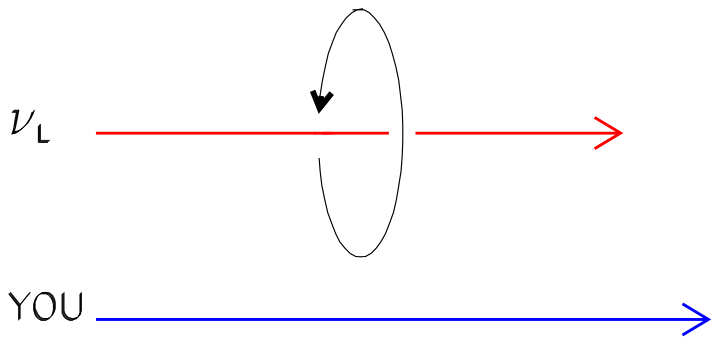

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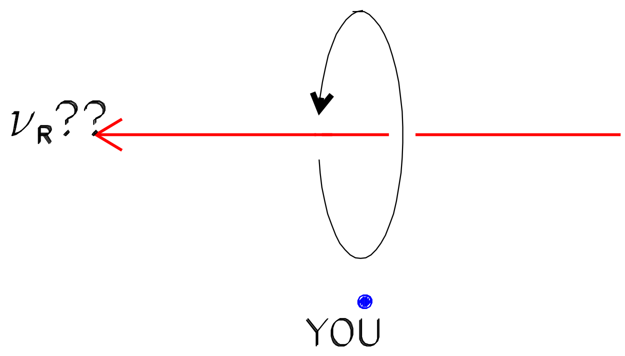

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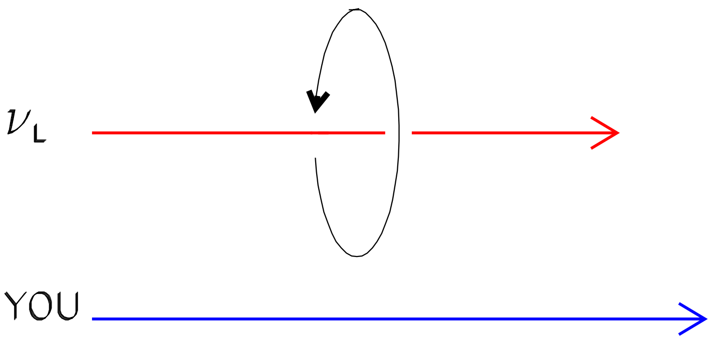

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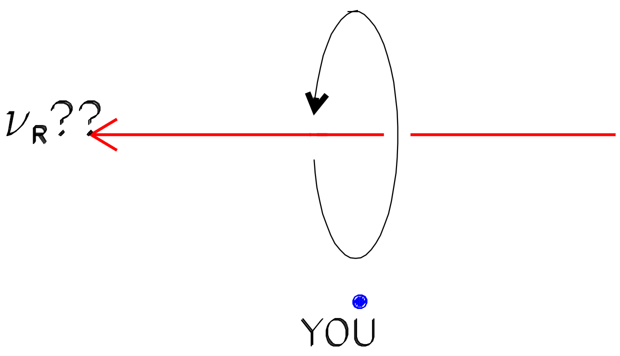

ν Mass from Non-Renormalizable Operator

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If SM is an effective low energy theory, for $E \ll \Lambda_{\rm NP}$

– The same particle content as the SM and same pattern of symmetry breaking

– But there can be non-renormalizable (dim>4) operators

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There is only one!

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which after symmetry breaking

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which after symmetry breaking

induces a ν Majorana mass

$$(M_\nu)_{ij} = Z^\nu_{ij} \frac{v^2}{\Lambda_{\rm NP}}$$

Implications:

– It is natural that ν mass is the first evidence of NP

– Naturally $m_\nu \ll$ other fermions masses $\sim \lambda^f v$ if $\Lambda_{\rm NP} >> v$

– See-saw with heavy ν_R integrated out is a particular example of this