Radiative corrections to $e^+e^- \to \pi^+\pi^-$ scattering

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In collaboration with

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60th International School of Subnuclear Physics

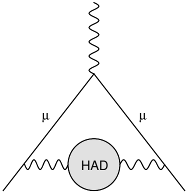
New Talent Session – 15 June 2024

UNIVERSITÀ DI PAVIA

INFN PAVIA

Hadronic contribution to muon g − 2

- The hadronic contribution to muon $g-2$ cannot be computed using perturbation theory
- The evaluation requires **data-driven** methods or **lattice** simulations
- The **time-like** (or **dispersive**) approach employs $\sigma\left(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons}\right)$ data
- Large uncertainty arising from the several resonances in the low-energy cross section

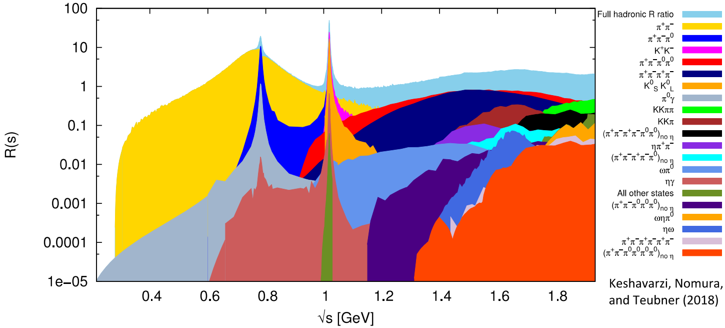


- Using the optical theorem and dispersive relations we obtain

$$a_\mu^{\mathrm{HLO}} \propto \int_{4m_\pi^2}^{\infty} \mathrm{ds}\, \frac{\mathrm{K(s)}\textcolor{red}{\mathrm{R(s)}}}{\mathrm{s}^2}$$

$$\textcolor{red}{\mathrm{R(s)}} = \frac{\sigma(\mathrm{e}^+\mathrm{e}^- \to \mathrm{hadrons})}{\sigma(\mathrm{e}^+\mathrm{e}^- \to \mu^+\mu^-)}$$

$$\mathrm{K(s)} \sim 1/\mathrm{s} \quad \to \quad \text{smooth}$$



Keshavarzi, Nomura, and Teubner (2018)

The $e^+e^- \to \pi^+\pi^-$ scattering

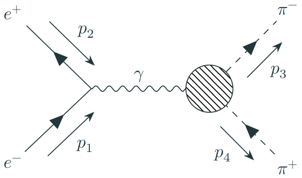
- $e^+e^- \to \pi^+\pi^-$ accounts for **72%** of a_μ^{had} and **78%** of δa_μ^{had}
- Pion **forward-backward asymmetry** w.r.t. initial e^- direction (θ):

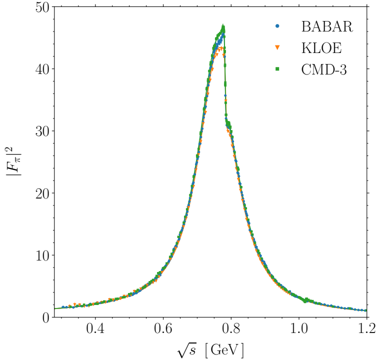
$$A_{FB}\left(\sqrt{s}\right) = \frac{\sigma_F(\cos\theta > 0) - \sigma_B(\cos\theta < 0)}{\sigma_F(\cos\theta > 0) + \sigma_B(\cos\theta < 0)}$$

- First direct measurement by the CMD-3 experiment, used for systematic error analysis
- Process implemented in the fully exclusive Monte Carlo event generator **BabaYaga@NLO** at NLO+PS with $m_e \neq 0$
- At leading order $\mathbf{d\sigma \propto |F_\pi|^2} \longrightarrow$ **Pion form factor:**

$$\langle \pi^\pm(p')|j^\mu_{\rm em}(0)|\pi^\pm(p)\rangle = \pm(p'+p)^\mu F_\pi\left((p'-p)^2\right) \times$$

- And beyond leading order? Three different possible approaches

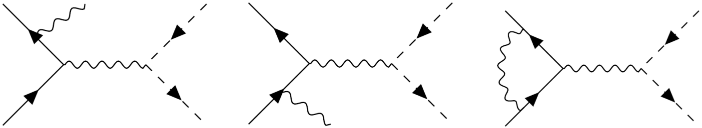




Factorised sQED approach: ISR and FSR

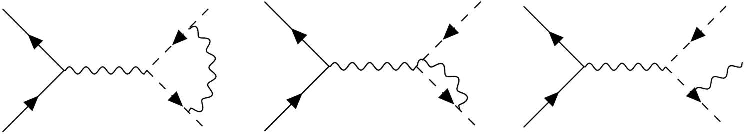
- NLO Initial-State Radiation (ISR):

$F_\pi(m_{\pi\pi}^2) \times$



- NLO Final-State Radiation (ISR):

$F_\pi(s) \times$

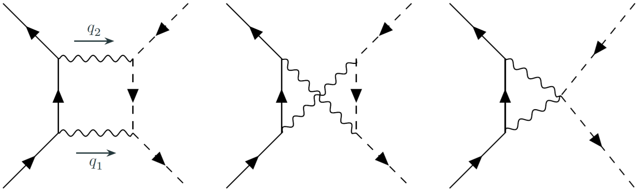


[1] G. Rodrigo, H. Czyz, J. H. Kuhn, M. Szopa, *Radiative return at NLO and the measurement of the hadronic cross-section in electron–positron annihilation*, Eur.Phys.J.C 24 (2002) 71-82

Factorised sQED approach: IFI

- NLO Initial-Final Interference (IFI):

$F_\pi(s) \;\times$



- The FsQED approach is justified because the IR divergence appears when

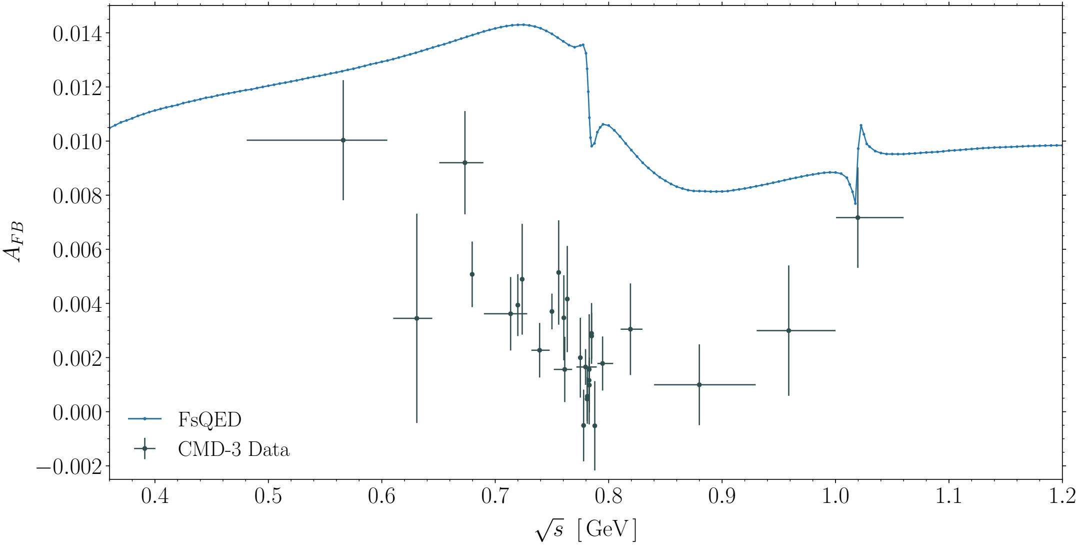
$$q_1 \to 0 \quad \Longrightarrow \quad F_\pi(q_1^2) \to 1 \quad F_\pi(q_2^2) \to F_\pi(s)$$

$$q_2 \to 0 \quad \Longrightarrow \quad F_\pi(q_2^2) \to F_\pi(s) \quad F_\pi(q_2^2) \to 1$$

- But this is valid only in the **soft limit** $\longrightarrow$ Is it enough?

- IFI is odd in $\cos\theta_\pm$ $\longrightarrow$ It induces the forward-backward asymmetry A_{FB}

Forward-backward asymmetry in BabaYaga@NLO: 1st try



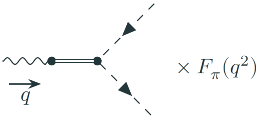
The GVMD approach

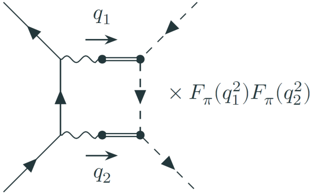
- Based on the **generalised vector meson dominance** introduced by Sakurai
- The form factor can be written as a sum of Breit-Wigner functions

$$F_\pi(q^2) = \sum_{v=0}^{N} c_v \frac{\Lambda_v^2}{\Lambda_v^2 - q^2} \qquad \text{with} \qquad \sum_{v=0}^{N} c_v = 1$$

$$\Lambda_v^2 = m_v^2 - i m_v \Gamma_v \qquad c_v = |c_v| e^{i\phi_v}$$

- **Pros**: writing $F_\pi(q^2)$ in a propagator-like form allows one to solve the loop integral with standard techniques without further approximations
- **Cons**: model-dependent and $\text{Im} F_\pi(q^2 < 4m_\pi^2) \neq 0 \longrightarrow$ breaks unitarity!

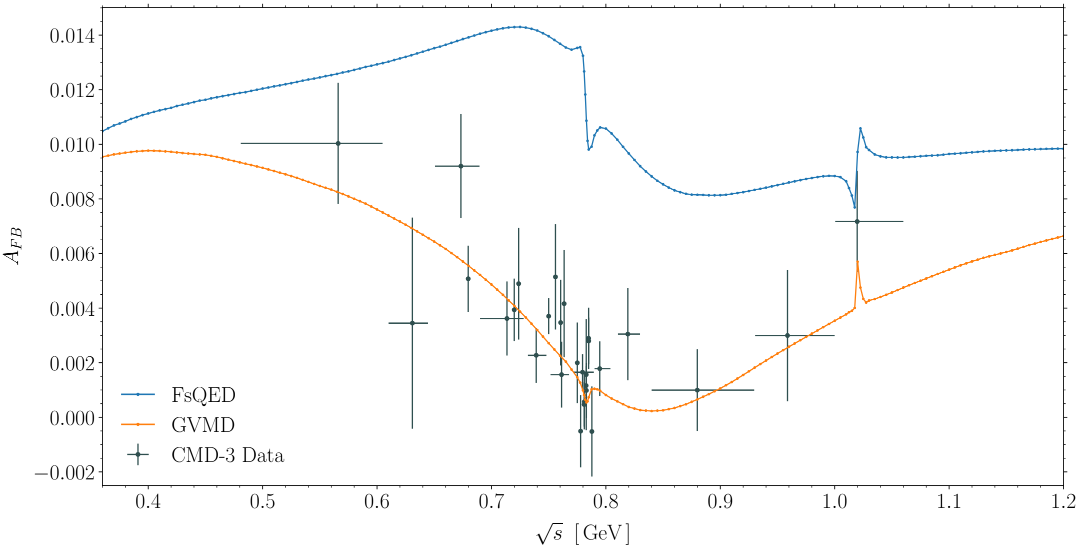




[2] J. J. Sakurai and D. Schildknecht, *Generalized vector dominance and inelastic electron-proton scattering, Phys. Lett. B 40 (1972) 121–126*

[3] F. Ignatov and R. N. Lee, *Charge asymmetry in $e^+e^- \to \pi^+\pi^-$ process*, Phys. Lett. B 833 (2022) 137283

Forward-backward asymmetry in BabaYaga@NLO: 2nd try



The dispersive approach

- Based on the general assumptions of **unitarity** and **analyticity**
- The form factor can be decomposed using the **dispersion relation**

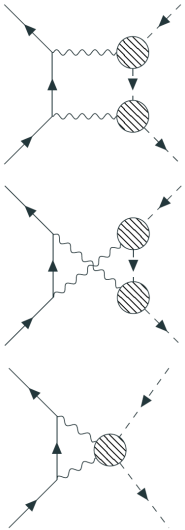
$$\frac{F_\pi(q^2)}{q^2} = \frac{1}{q^2-\lambda^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\mathrm{Im}F_\pi(s')}{s'(q^2-s')}$$

- In terms of a finite photon mass $\lambda \to 0$, the IFI matrix element is splitted as

$$\mathcal{M}_{IFI}^{disp} = \mathcal{M}_{IFI}^{point}(\lambda^2,\lambda^2) \qquad \text{pole-pole}$$

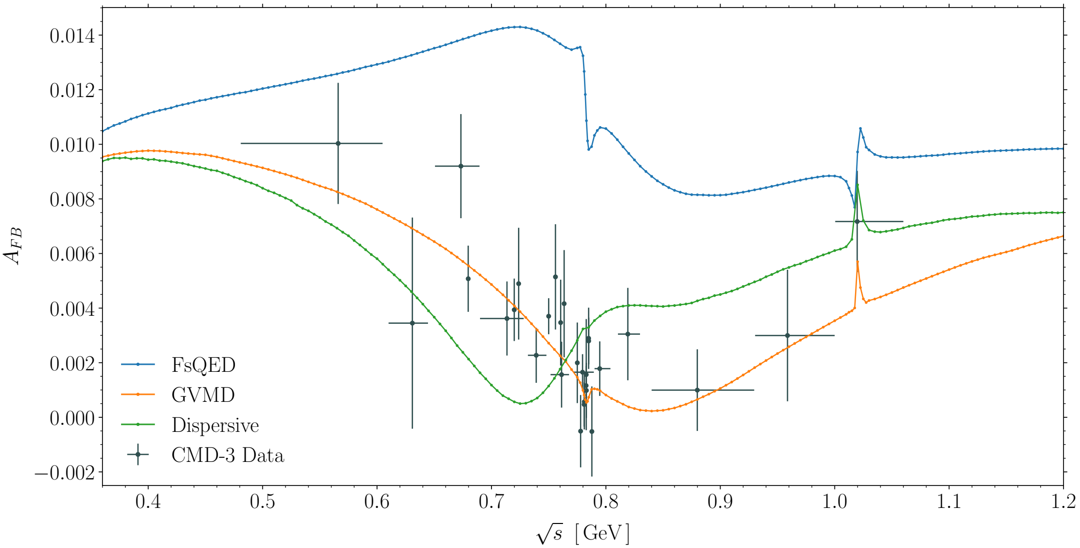
$$-\frac{1}{\pi}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\mathrm{Im}F_\pi(s')\left[\mathcal{M}_{IFI}^{point}(s',\lambda^2)+\mathcal{M}_{IFI}^{point}(\lambda^2,s')\right] \qquad \text{pole-disp}$$

$$+\frac{1}{\pi^2}\int_{4m_\pi^2}^{\infty}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'}\frac{ds''}{s''}\mathrm{Im}F_\pi(s')\mathrm{Im}F_\pi(s'')\mathcal{M}_{IFI}^{point}(s',s'') \qquad \text{disp-disp}$$



[4] G. Colangelo, M. Hoferichter, J. Monnard, and J. R. de Elvira, *Radiative corrections to the forward-backward asymmetry in $e^+e^- \to \pi^+\pi^-$*, JHEP 08 (2022) 295

Forward-backward asymmetry in BabaYaga@NLO: 3rd try



An ongoing community effort

Coming soon: *Radiative corrections and Monte Carlo tools for low-energy hadronic cross sections in e^+e^- collisions*

Backup

Muon g−2 anomaly