Studying the low-energy regime of QCD

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QCD at low energies

QCD has a very simple Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4g^2}\text{Tr}[F_{\mu\nu}F^{\mu\nu}] + \sum_f \overline{q}_f(i\gamma_\mu D^\mu - m_f)q_f$$

Asymptotic freedom ⟶ Perturbative methods fail at low energies

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- Large N_c limit
- Chiral perturbation theory
- Lattice QCD

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Asymptotic freedom ⟶ Perturbative methods fail at low energies

- Large N_c limit
- Chiral perturbation theory ⟶ **$\pi\pi$ scattering as a function of N_c**
- Lattice QCD

Meson-meson scattering

$$N_f = 4$$
$$(m_u = m_d = m_s = m_c)$$

→

Degenerate ~~mesons~~ pions
$$M_\pi = M_K = M_D = M_\eta$$

7 scattering channels

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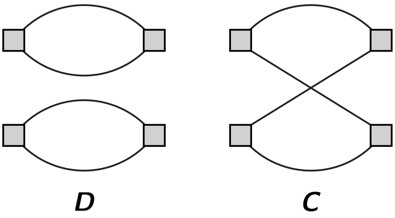
7 scattering channels

$$15 \otimes 15 = \mathbf{84\,(SS)} \oplus 45 \oplus 45 \oplus \mathbf{20\,(AA)} \oplus 15 \oplus 15 \oplus 1$$

$\pi^+\pi^+$ $\quad$ $D_s^+\pi^+ - D^+K^+$

$C_{SS} = 2(D - C)$

$C_{AA} = 2(D + C)$



Large N_c limit

Large N_c ($N_c \to \infty$, $N_f = \text{const}$) keeps non-perturbative features of QCD

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We can characterize the N_c and N_f scaling of diagrams at large N_c

- Color loops $\sim N_c$
- Vertex $\sim g \sim N_c^{-1/2}$
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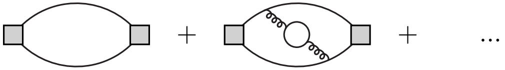
$C_\pi =$ 

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$= \quad aN_c$

$a, b \sim \mathcal{O}(1)$ constants

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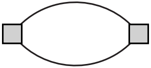
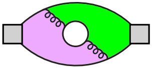
$C_\pi =$  $+$  $+ \quad \dots$

$= \quad aN_c \quad + \quad bg^4 N_c^2 N_f$

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$$C_\pi = \quad + \quad + \quad \dots$$

$$= aN_c + bg^4 N_c^2 N_f + \mathcal{O}(N_c^{-1})$$

$$= bN_f$$

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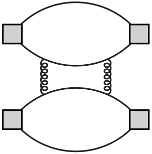
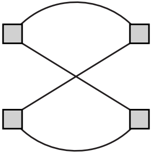
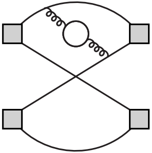
Scattering at large N_c

$c, d, e, f \sim \mathcal{O}(1)$ constants

Two pions:

$D = \quad C_\pi^2 \quad +$  $+ \quad \ldots \quad = C_\pi^2 + d + \mathcal{O}(N_c^{-1})$

$C =$  $+$  $+ \quad \ldots \quad = eN_c + fN_f + \mathcal{O}(N_c^{-1})$

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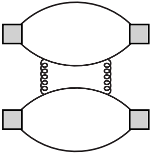

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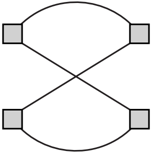

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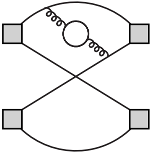

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Scattering amplitude in the *SS* and *AA* channels:

$$\mathcal{M}^{SS,AA} = \mp \frac{1}{N_c}\left(\tilde{a} + \tilde{b}\frac{N_f}{N_c} \pm \tilde{c}\frac{1}{N_c}\right) + \mathcal{O}(N_c^{-3})$$

Same scaling for other scattering observables

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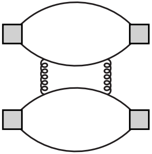
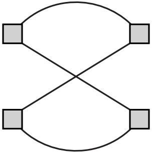
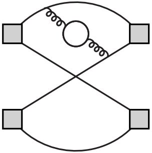
$D = \quad C_\pi^2 \quad +$  $+ \quad \ldots \quad = C_\pi^2 + d + \mathcal{O}(N_c^{-1})$

$C =$  $+$  $+ \quad \ldots \quad = e N_c + f N_f + \mathcal{O}(N_c^{-1})$

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Chiral Perturbation Theory (ChPT)

ChPT describes QCD in terms of pseudo-Goldstone bosons (**pions**)

$$\phi = \begin{pmatrix} \pi^0 + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}\pi^+ & \sqrt{2}K^+ & \sqrt{2}D^0 \\ \sqrt{2}\pi^- & -\pi^0 + \frac{\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}K^0 & \sqrt{2}D^+ \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2\eta_0}{\sqrt{3}} + \frac{\eta_c}{\sqrt{6}} & \sqrt{2}D_s^+ \\ \sqrt{2}\bar{D}^0 & \sqrt{2}D^- & \sqrt{2}D_s^- & -\frac{3\eta_c}{\sqrt{6}} \end{pmatrix} \qquad (N_f = 4)$$

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Most general lagrangian with QCD symmetries

$$U = e^{i\phi/F}$$

$$\mathcal{L}_2 = \frac{F^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{F^2 B_0}{2}\mathrm{Tr}[\chi U^\dagger + \chi^\dagger U] \qquad \text{(2 LECs)}$$

$$F^2 \sim \mathcal{O}(N_c)$$

$$B_0, M_\pi \sim \mathcal{O}(1)$$

$$\mathcal{L}_4 = \sum_{i=0}^{12} L_i O_i \qquad L_i \sim \mathcal{O}(N_c) \text{ or } \mathcal{O}(1) \qquad \text{(13 LECs)}$$

ChPT at large N_c

The η' needs to be included

$$M_{\eta'}^2 = M_\pi^2 + \frac{2N_f\chi_{top}}{F_\pi^2} \xrightarrow[\text{large } N_c]{F_\pi^2 \sim \mathcal{O}(N_c)} M_\pi^2 + \dots$$

[Witten-Veneciano]

Large N_c or U(N_f) ChPT [Kaiser, Leutwyler 2000]:

- Include η' in pion matrix

$$\phi|_{U(N_f)} = \phi|_{SU(N_f)} + \eta' \mathbb{1}$$

- Leutwyler counting scheme

$$\mathcal{O}(m_q) \sim \mathcal{O}(M_\pi^2) \sim \mathcal{O}(k^2) \sim \mathcal{O}(N_c^{-1})$$

$\pi\pi$ scattering in ChPT

$\pi\pi$ **scattering at LO in ChPT** [Weinberg 1979]

$$k \cot \delta_0 = \frac{1}{a_0} + \dots$$

$$M_\pi a_0^{SS} = -\frac{M_\pi^2}{16\pi F_\pi^2} \propto -\frac{1}{N_c}$$

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$\pi\pi$ **scattering at NNLO in large N_c ChPT** [JBB et al. 2022]

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Large N_c $\longrightarrow$

$$L_{SS} = \underbrace{N_c L^{(0)} + N_f L_c^{(1)}}_{\text{Same sign}} \underbrace{- L_a^{(1)}}_{\text{Opposite sign}} + \mathcal{O}(N_c^{-1})$$

$$L_{AA} = N_c L^{(0)} + N_f L_c^{(1)} + L_a^{(1)} + \mathcal{O}(N_c^{-1})$$

Same sign — Opposite sign

QCD in the lattice

Lattice QCD is a first-principles approach to the strong interaction

$$\langle O[\phi] \rangle = \frac{1}{Z} \int \mathcal{D}\phi \, e^{iS[\phi]} O[\phi]$$

$\phi \equiv$ quark, gluons
$S[\phi] \equiv$ QCD action
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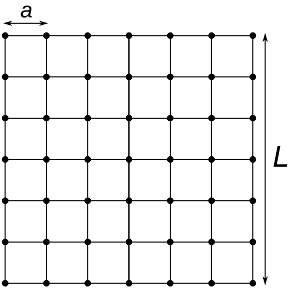
Finite, discretized spacetime

+

Monte-Carlo methods



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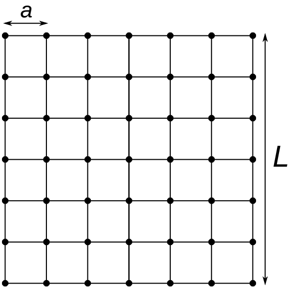

Allows to simulate QCD for varying N_c, M_π, ...

Meson-meson scattering in the lattice

Particle scattering cannot be directly studied in the lattice

Scattering	**Lattice QCD**
Real-time process	Euclidean time
Infinite volume	Finite volume
Asymptotic states	Stationary states

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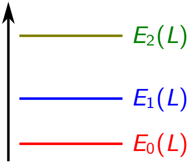

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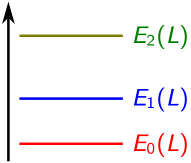
Infinite-volume scattering amplitudes

Quantization condition (QC) ⟷



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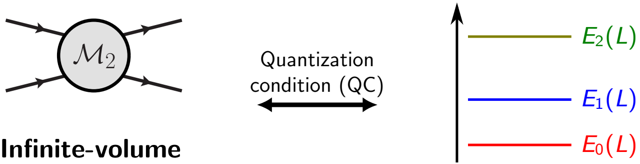


Infinite-volume scattering amplitudes

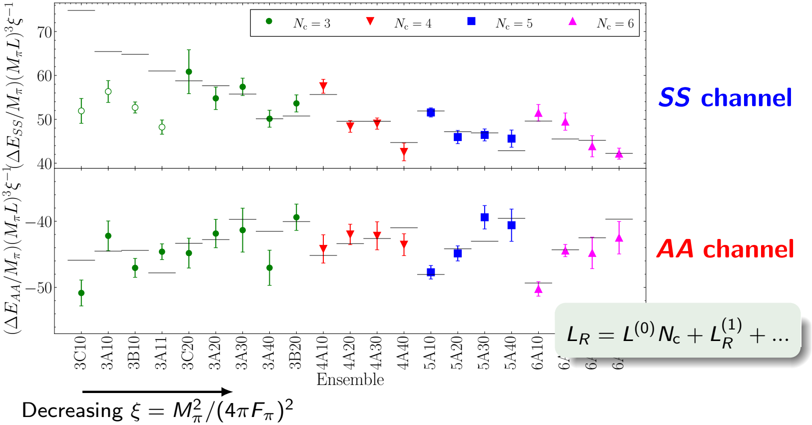
Finite-volume spectrum

Two particles in *s*-wave:

$$k \cot \delta_0 = \frac{2}{\gamma L \pi^{1/2}} \mathcal{Z}_{00}^{\boldsymbol{P}}\left(\frac{kL}{2\pi}\right) \xrightarrow[\text{energies}]{\text{Low}} E_0 - 2m = -\frac{4\pi a_0}{M_\pi L^3} + \ldots$$

Fit to large N_c ChPT

Chiral and N_c fit of both channels to U(4) ChPT [JBB et al. 2022],



$$\frac{L_{SS,AA}}{N_c} \times 10^3 = -0.02(8) - 0.01(5)\frac{N_f}{N_c} \mp 1.76(20)\frac{1}{N_c}$$

Summary and outlook

We can study the non-perturbative regime of QCD combining the **large N_c limit**, **ChPT** and **lattice QCD**

- We are studying the large N_c scaling of scattering observables
- We have found large **subleading N_c effects**

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Ongoing work: possible tetraquark resonance in AA channel

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Thank you for your attention!