

# Exploring dense nuclear matter with neutron stars

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UNIVERSITY



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Prof. Jürgen Schaffner-Bielich, Goethe Universität Frankfurt  
Prof. Bence Kocsis, University of Oxford

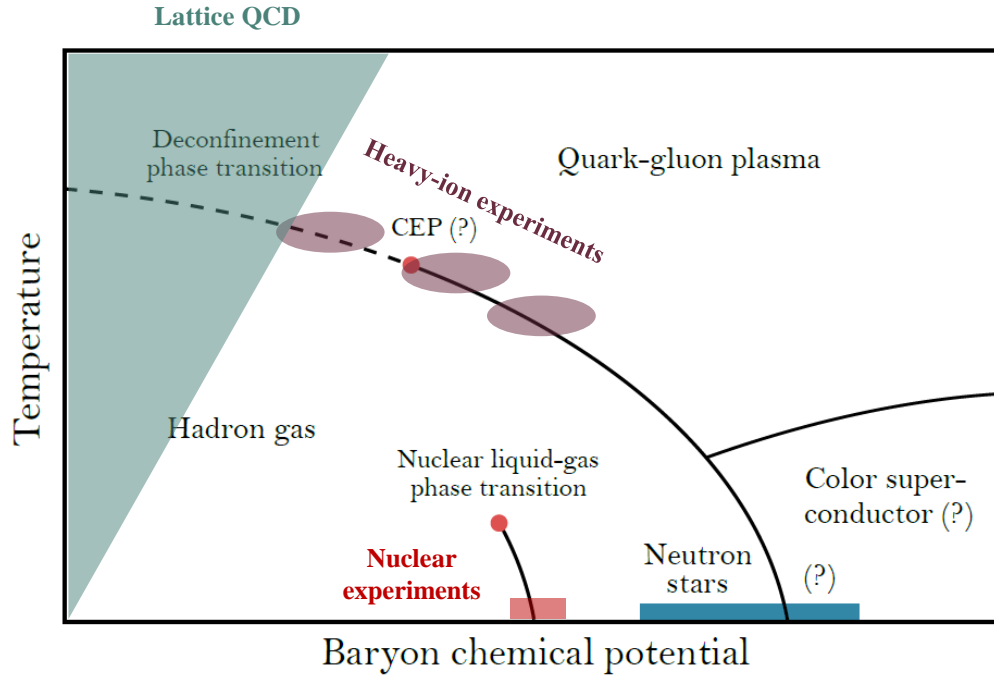


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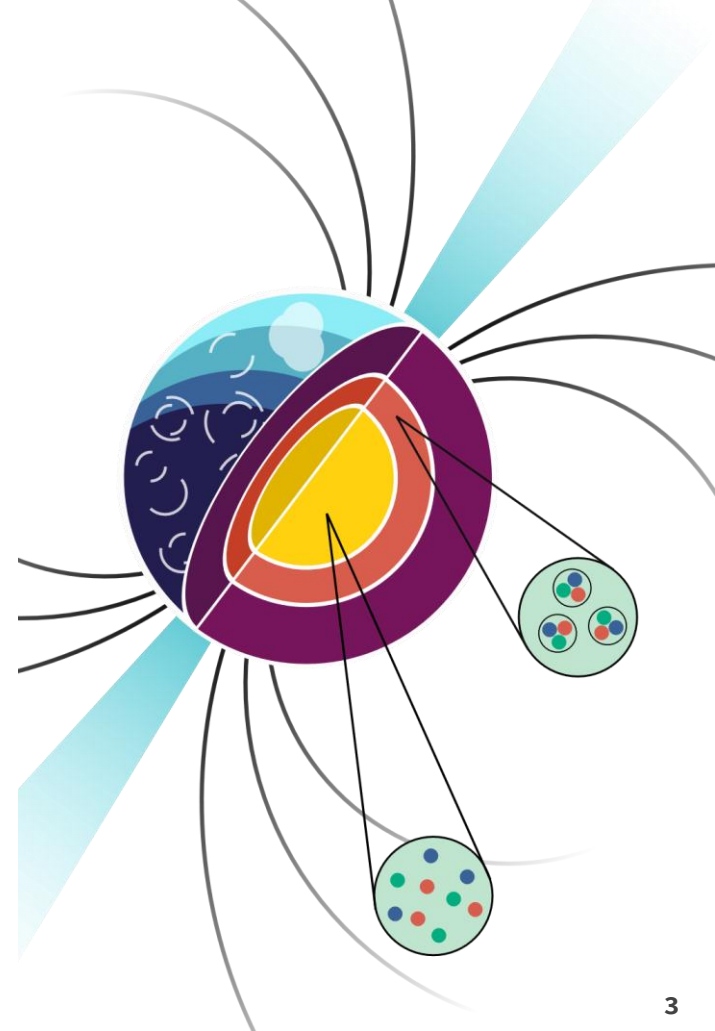
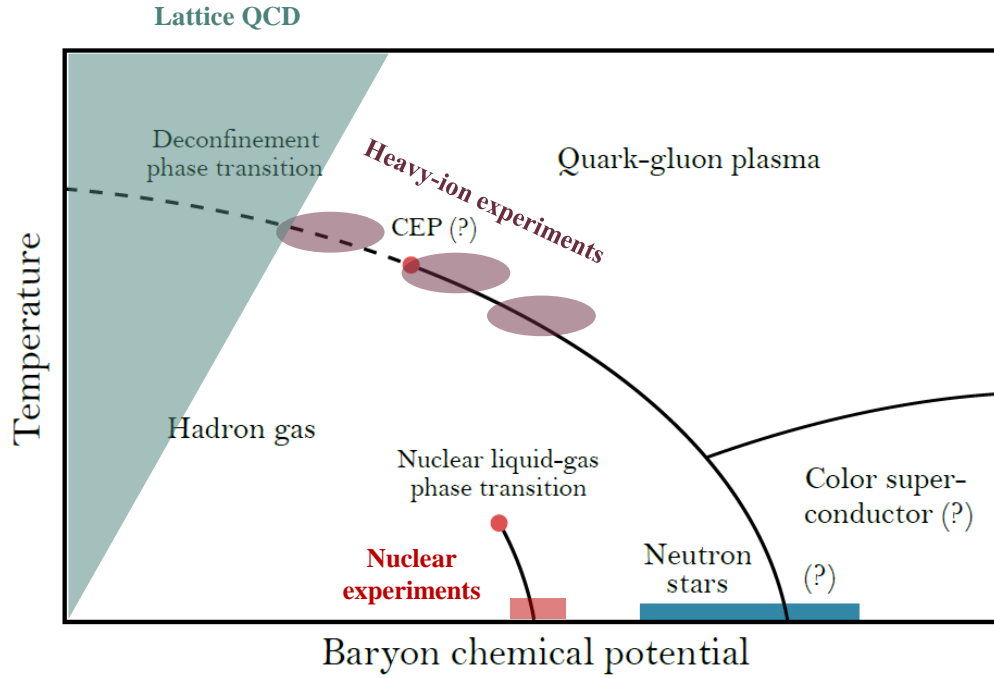
**International School of Subnuclear Physics, Erice, 2024.06.17.**

# Why study neutron stars?

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# The mass-radius relation

$$\frac{dp}{dr} = -[\varepsilon(r) + p(r)] \frac{M(r) + 4\pi r^3 p(r)}{r^2 - 2M(r)r}$$

How to get a mass-radius relation:

→ get an **equation of state**  $p(\varepsilon)$

→ start with a **specific central density**:  $\varepsilon_c$ ,  $p_c$ ,  $M(0) = 0$

→ integrate the TOV equations until  $p(R) = 0$  → **R is the radius** of the NS

→ **M(R) is the mass** of the NS

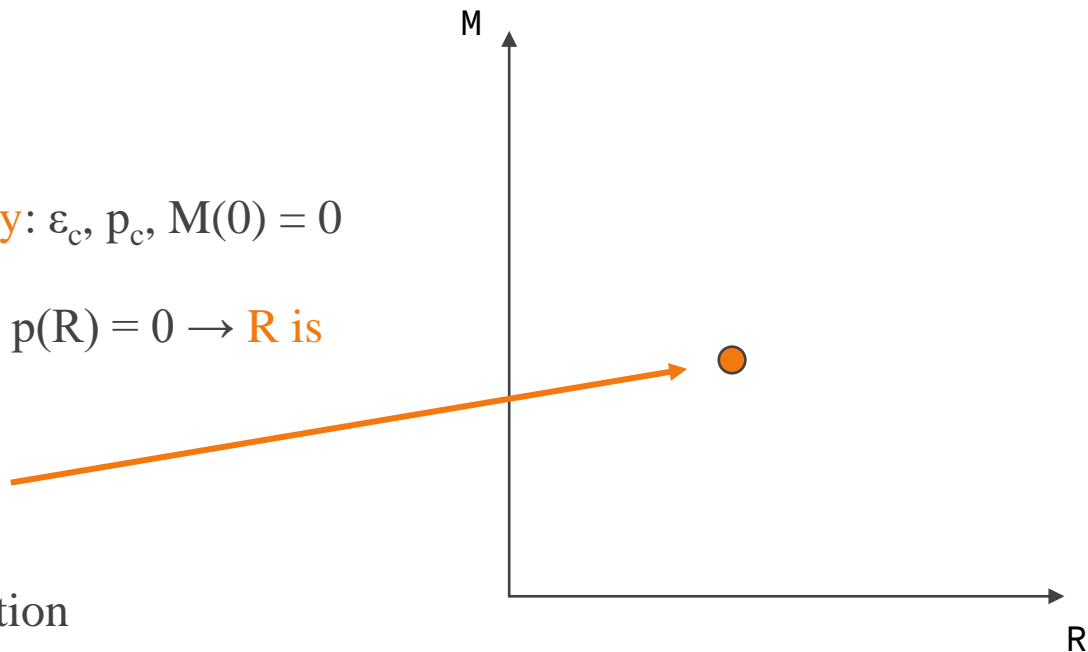
→ change  $\varepsilon_c$  and repeat → M-R relation

# The mass-radius relation

$$\frac{dp}{dr} = -[\varepsilon(r) + p(r)] \frac{M(r) + 4\pi r^3 p(r)}{r^2 - 2M(r)r}$$

How to get a mass-radius relation:

- get an **equation of state**  $p(\varepsilon)$
- start with a **specific central density**:  $\varepsilon_c, p_c, M(0) = 0$
- integrate the TOV equations until  $p(R) = 0 \rightarrow R$  is **the radius** of the NS
- **$M(R)$  is the mass** of the NS
- change  $\varepsilon_c$  and repeat  $\rightarrow$  M-R relation

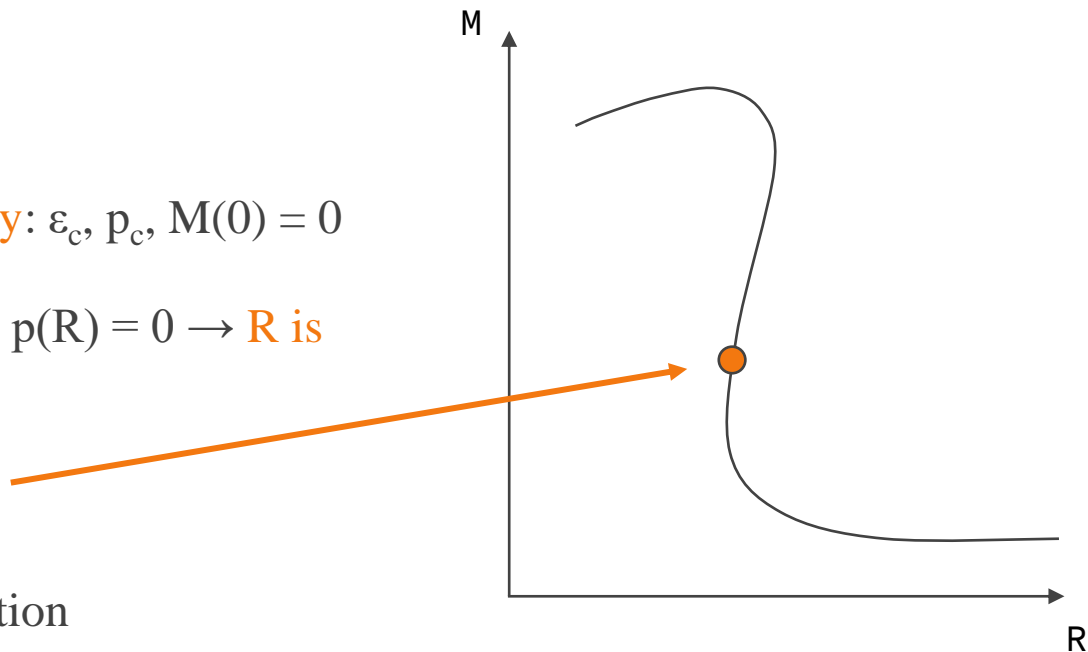


# The mass-radius relation

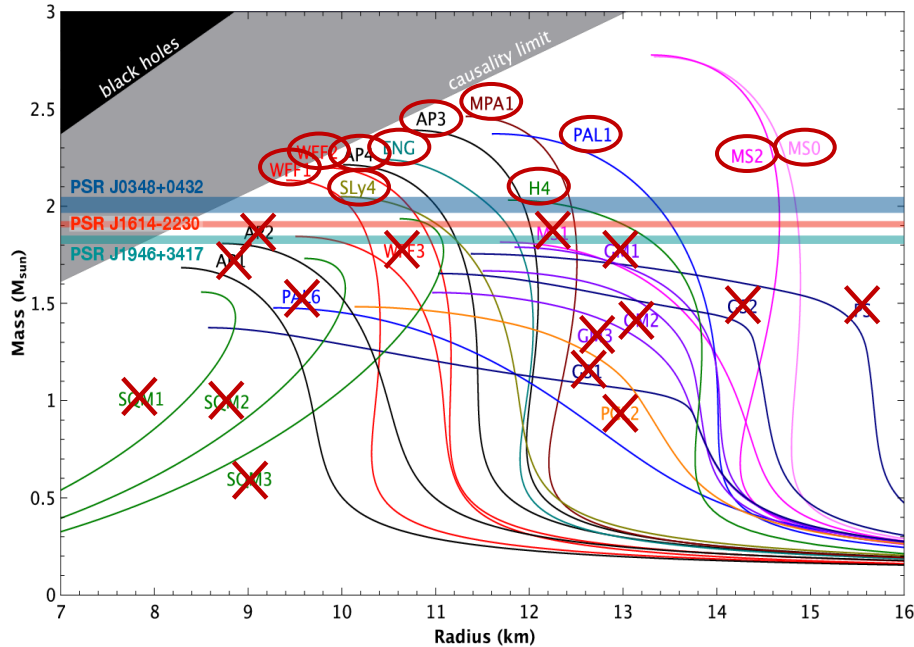
$$\frac{dp}{dr} = -[\varepsilon(r) + p(r)] \frac{M(r) + 4\pi r^3 p(r)}{r^2 - 2M(r)r}$$

How to get a mass-radius relation:

- get an **equation of state**  $p(\varepsilon)$
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- **$M(R)$  is the mass** of the NS
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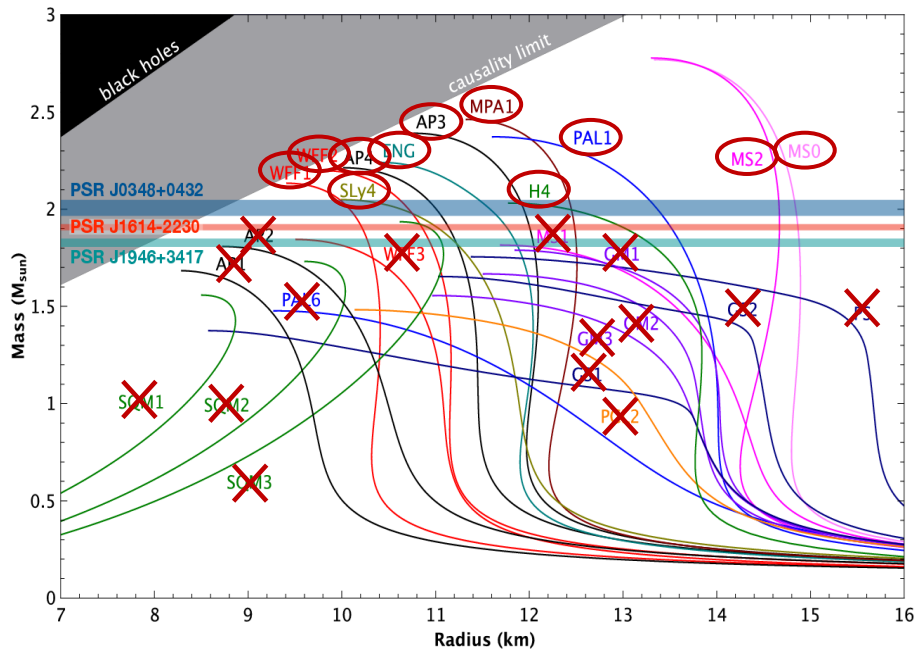


# Astrophysical measurements

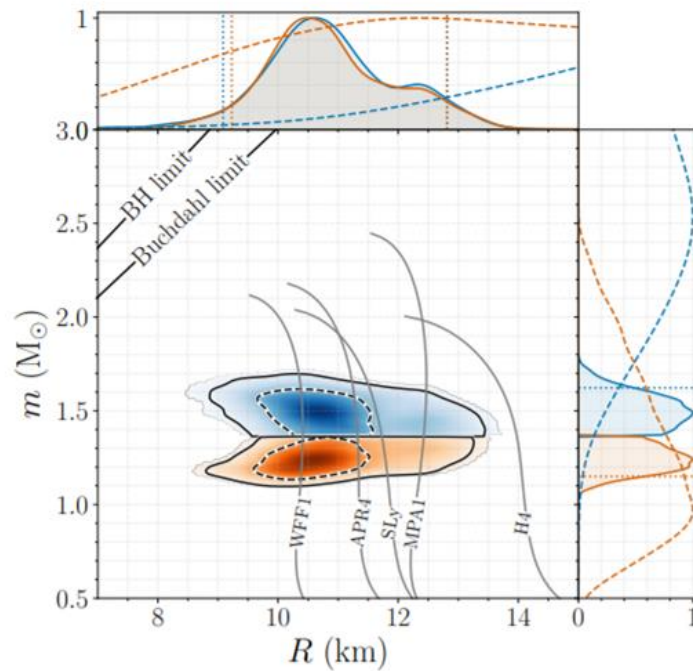
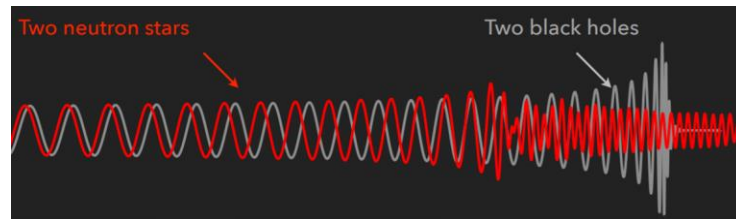


Credits: Norbert Wex

# Astrophysical measurements



Credits: Norbert Wex



Source: LVC, *Phys.Rev.Lett.* 121, 161101 (2018)



# Hybrid stars

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Hybrid stars also have a **hadronic crust and quark matter core**:

- at low densities we use hadronic EoSs:
  - ↪ the **SFHo** EoS as a soft hadronic EoS
  - ↪ the **DD2** as a stiff EoS
- effective **quark-meson model** consistent with meson phenomenology and lattice QCD
- we connect the two phases smoothly:
  - ↪  **$\varepsilon(n_B)$  interpolation** with polynomial

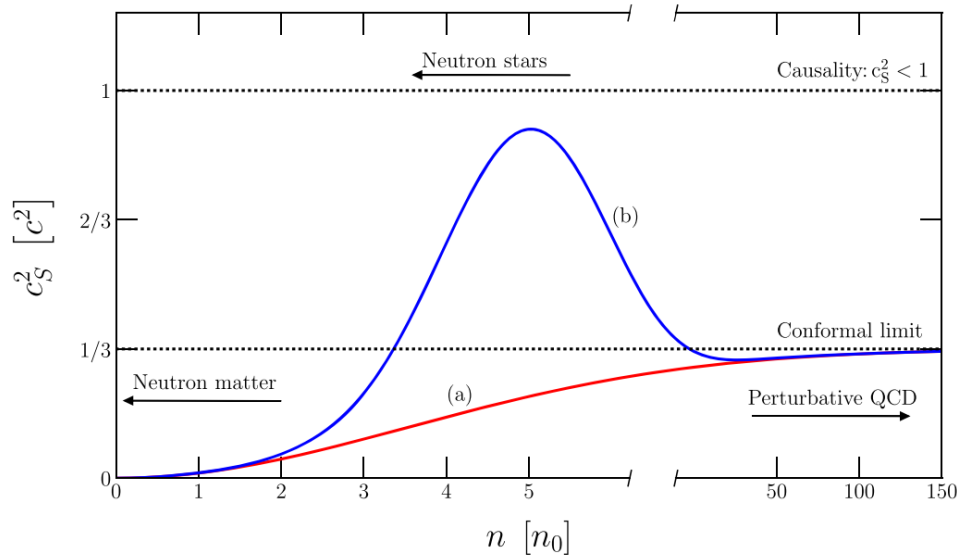
Observations → **Bayesian statistics**

$$p(\vartheta|\text{data}) = \frac{p(\text{data}|\vartheta)p(\vartheta)}{p(\text{data})}$$

$$p(\text{data}|\vartheta) = p(M_{\max}|\vartheta)p(\text{NICER}|\vartheta)p(\tilde{\Lambda}|\vartheta)$$

- ↪ Lower limit on maximum mass from **massive pulsars**
- ↪ **NICER** mass-radius constraints
- ↪ **Tidal deformability** measurement from **GW170817**
- ↪ Upper mass constraint from **hypermassive neutron star hypothesis**

# Speed of sound and conformality



Source: I. Tews, et al. In: *Astrophys.J.* 860, 149 (2018)

Important measure: **speed of sound**

$$c_s^2 = \frac{dp}{d\varepsilon}$$

In the **conformal limit** (high density):

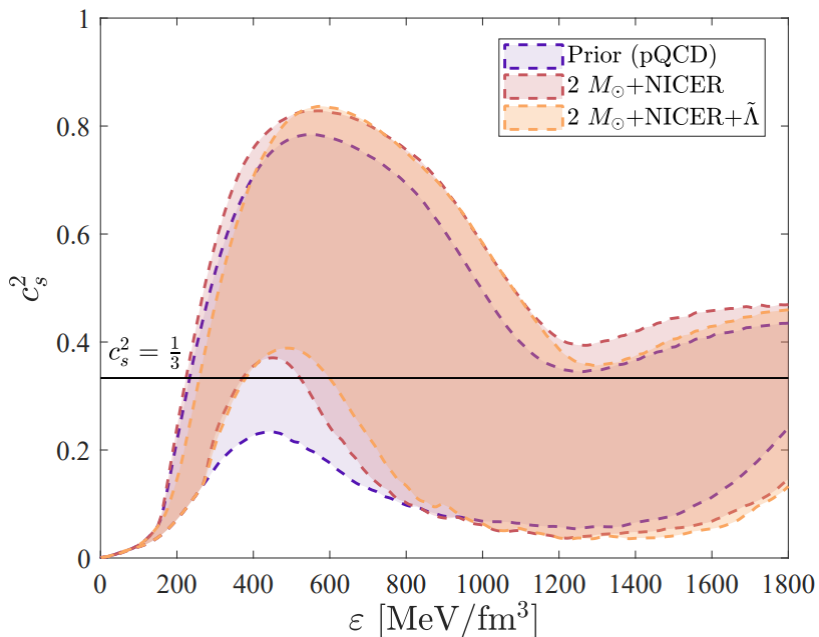
$$p \rightarrow \frac{1}{3}\varepsilon \quad c_s^2 \rightarrow \frac{1}{3}$$

Empirical **conformality measures**:

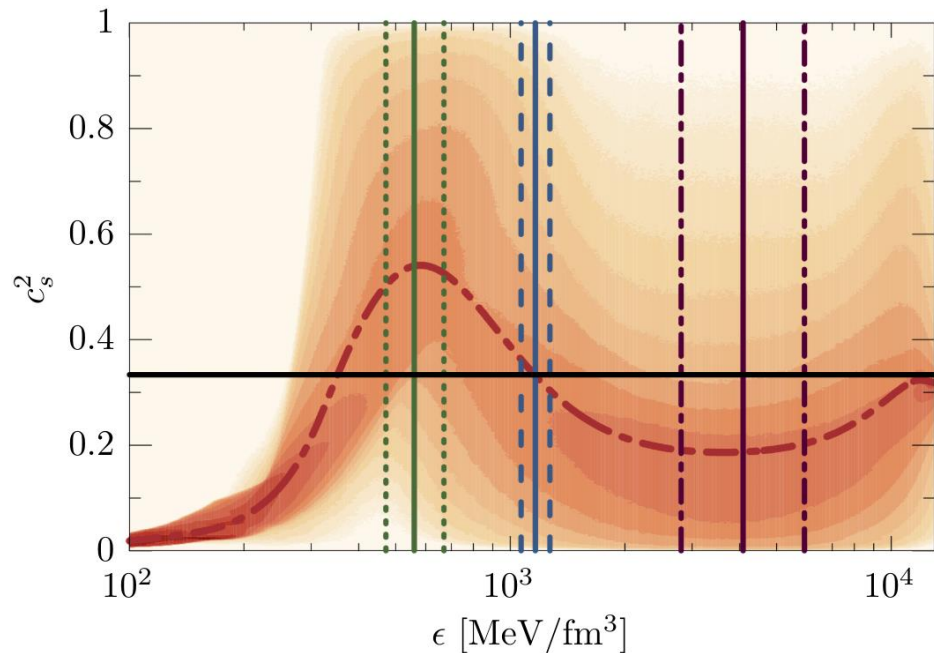
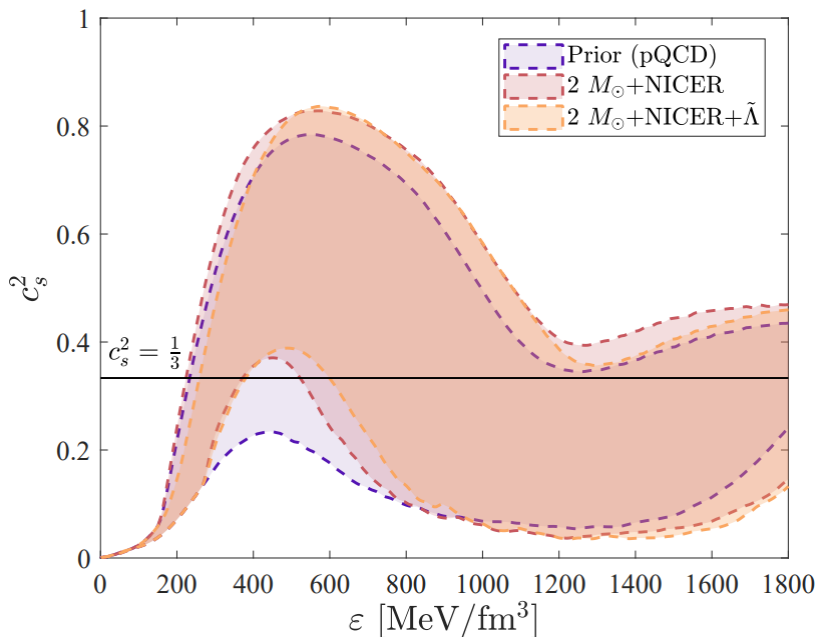
$$\Delta = \frac{1}{3} - \frac{p}{\varepsilon} \quad d_c = \sqrt{\Delta^2 + \Delta'^2}$$

# Speed of sound and conformality

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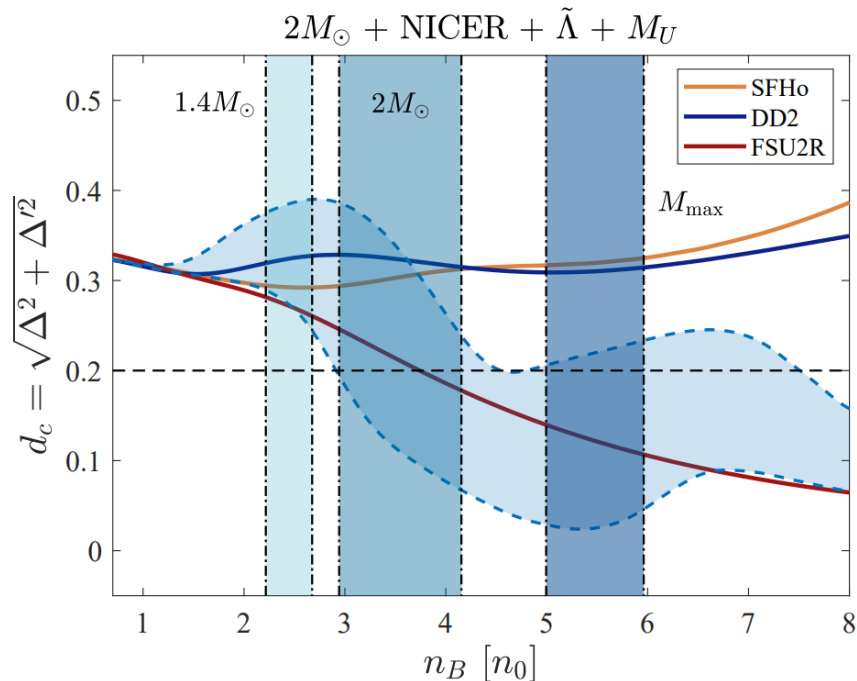
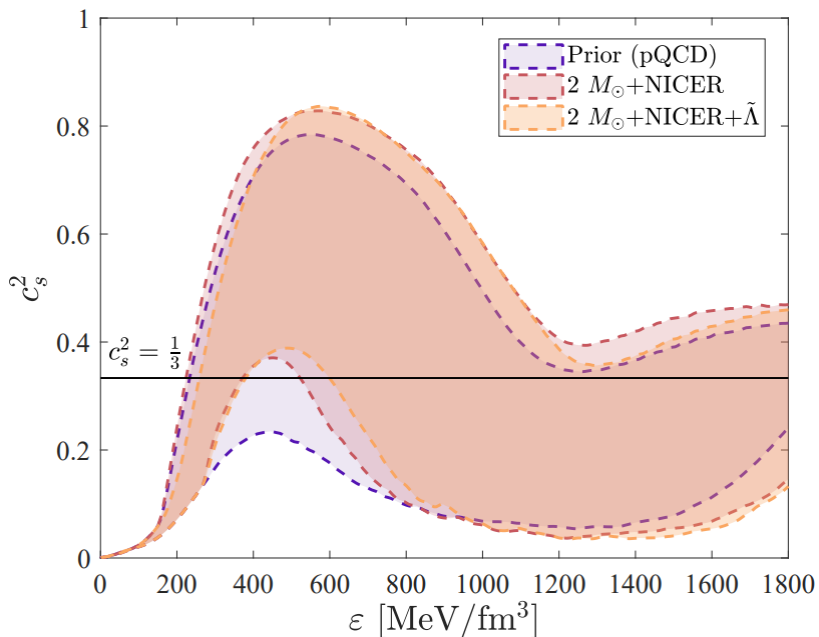


# Speed of sound and conformality



Source: M. Marczenko, et al., Phys.Rev.C 107, 025802 (2023)

# Speed of sound and conformality



Source: J. Takátsy, et al. In: *Phys.Rev.D* 108, 043002 (2023)

**Thank you for your attention!**

# References to some of our works

— — —

- [1] P. Kovács, Zs. Szép, Gy. Wolf. In: Phys.Rev.D 93, 114014 (2016)
- [2] J. Takátsy, P. Kovács. In: Phys.Rev.D 102, 028501 (2020)
- [3] P. Kovács, J. Takátsy, J. Schaffner-Bielich, Gy. Wolf. In: Phys.Rev.D 105, 103014 (2022)
- [4] P. Kovács, Gy. Kovács, F. Giacosa. In: Phys.Rev.D 106, 116016 (2022)
- [5] J. Takátsy, P. Kovács, Gy. Wolf, J. Schaffner-Bielich. In: Phys.Rev.D 108, 043002 (2023)
- [6] Gy. Kovács, P. Kovács, Gy. Wolf, P. M. Lo, K. Redlich. In: Phys.Rev.D 108, 076010 (2023)

**Backup slides**



# The quark-meson model

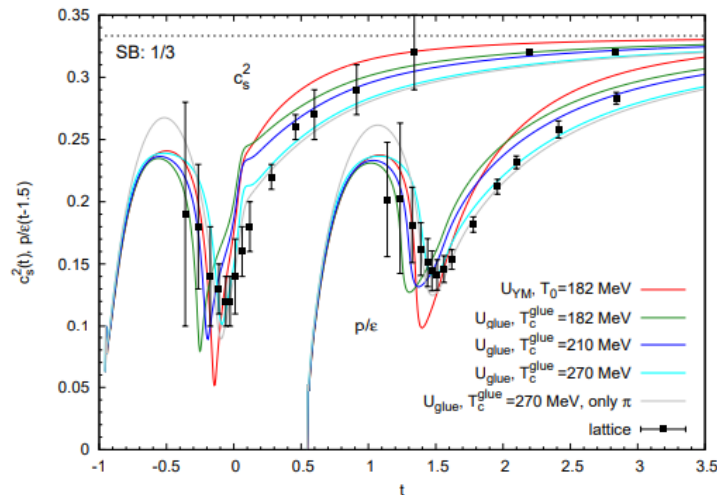
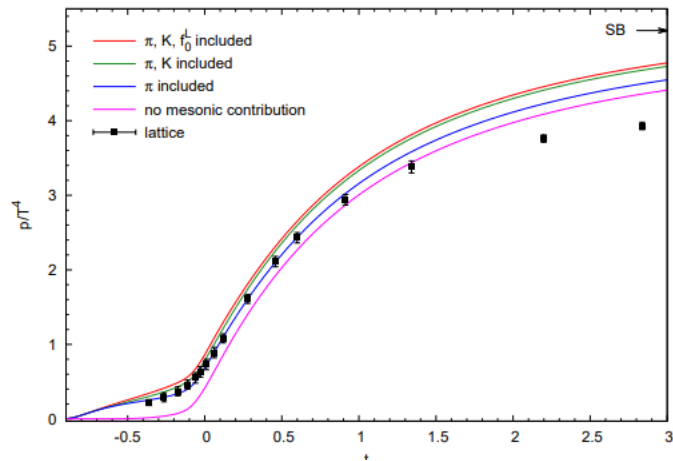
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We use the (axial)vector meson extended linear sigma model

↪ three-flavour constituent quark-meson model with the complete (pseudo)scalar and (axial)vector meson nonets

↪ parameterized with meson vacuum masses and decay widths

↪ agrees well with lattice results at finite temperature



# The eLSM Lagrangian

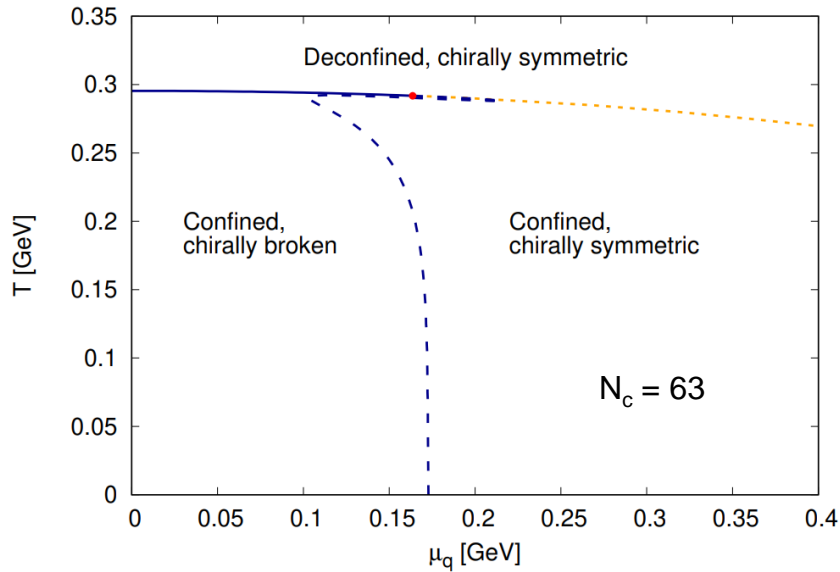
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The eLSM Lagrangian is constructed based on linearly realized **global**  $U(3)_L \times U(3)_R$  **chiral** **symmetry** and its **explicit breaking**

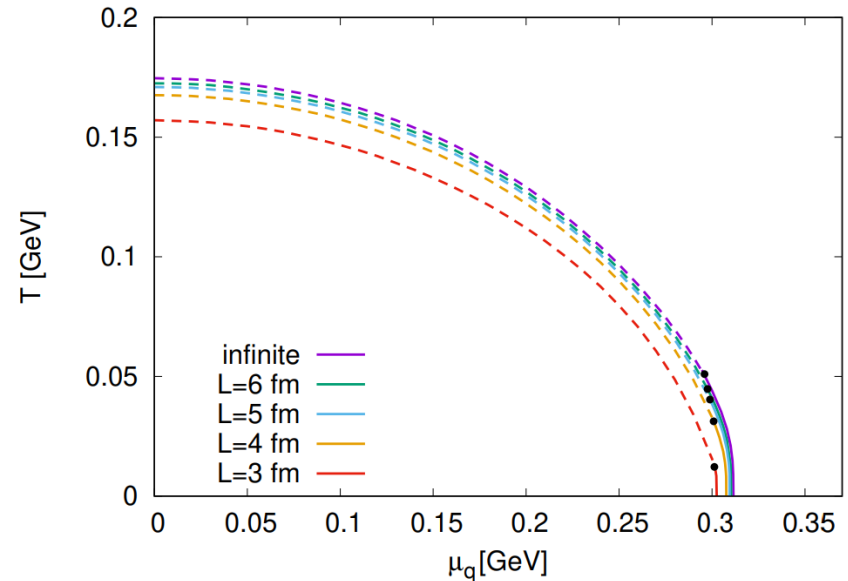
$$\begin{aligned} \mathcal{L} = & \text{Tr}[(D_\mu \Phi)^\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & + c_1 (\det \Phi + \det \Phi^\dagger) + \text{Tr}[H(\Phi + \Phi^\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\ & + \text{Tr} \left[ \left( \frac{m_1^2}{2} \mathbb{I} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} (\text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\}) \\ & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) \\ & + \bar{\Psi} i \gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (\Phi_S + i \gamma_5 \Phi_{PS}) \Psi - g_V \bar{\Psi} (\gamma^\mu (V_\mu + \gamma_5 A_\mu)) \Psi \end{aligned}$$

$$\begin{aligned} D^\mu \Phi &= \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA_e^\mu [T_3, \Phi], \\ L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu [T_3, L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu [T_3, L^\mu]\}, \\ R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu [T_3, R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu [T_3, R^\mu]\}, \\ D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with } G^\mu = g_s G_a^\mu T_a. \end{aligned}$$

# Large- $N_c$ limit and finite volume effects



Source: P. Kovács, et al. In: *Phys.Rev.D* 106, 116016 (2022)



Source: Gy. Kovács, et al. In: *Phys.Rev.D* 108, 076010 (2023)

↪ large- $N_c$  solution can be **very different** from  $N_c=3$  solution

↪ the **CEP** temperature is reduced for finite-size systems

# Mass measurement

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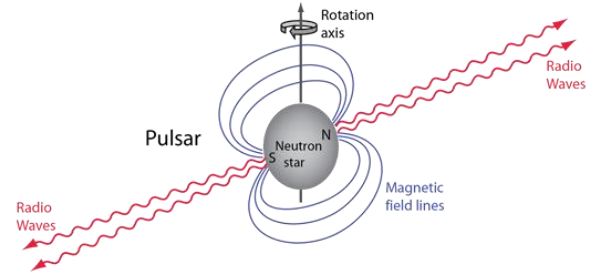
→ Pulsars in **binary systems** + Doppler shift

→ Degeneracies → only projected semi-major axis is measured

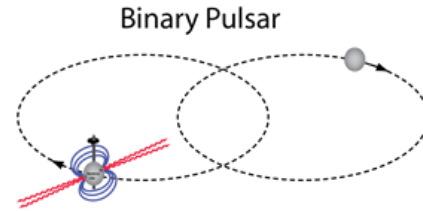
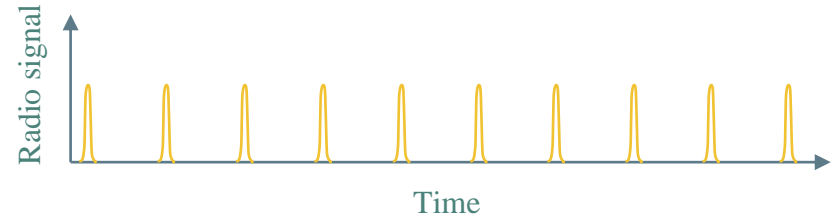
$$\left(\frac{2\pi}{P_{\text{orb}}}\right)^2 \frac{(a_{\text{ns}} \sin i)^3}{G} = \frac{(M_{\text{c}} \sin i)^3}{M_{\text{T}}^2}$$

$$q = \frac{M}{M_{\text{c}}} = \frac{(a_{\text{c}} \sin i)^3}{(a_{\text{ns}} \sin i)^3}$$

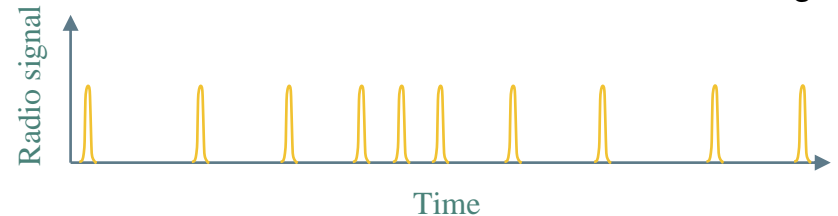
→ Observation of the **companion object**:  
another pulsar, white dwarf (+X-ray binaries)



Credits: HyperPhysics



Credits: HyperPhysics



# Mass measurement

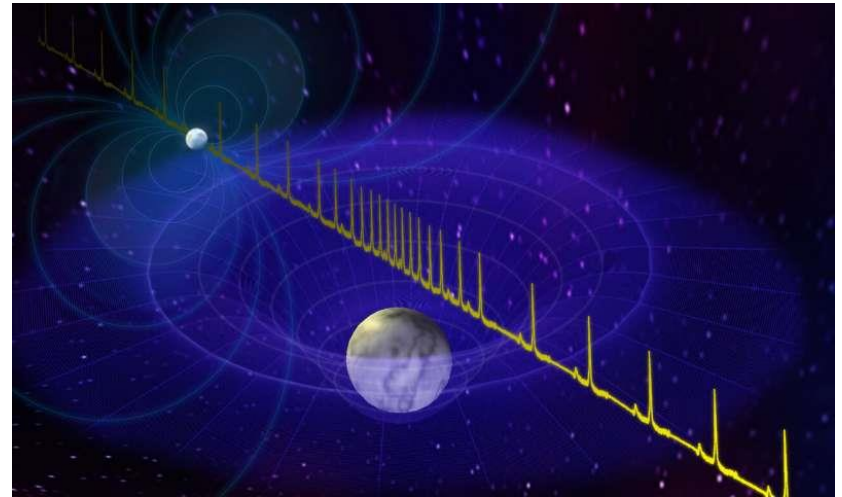
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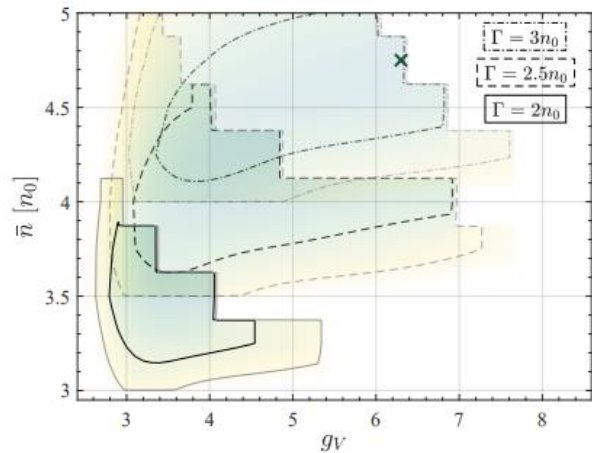
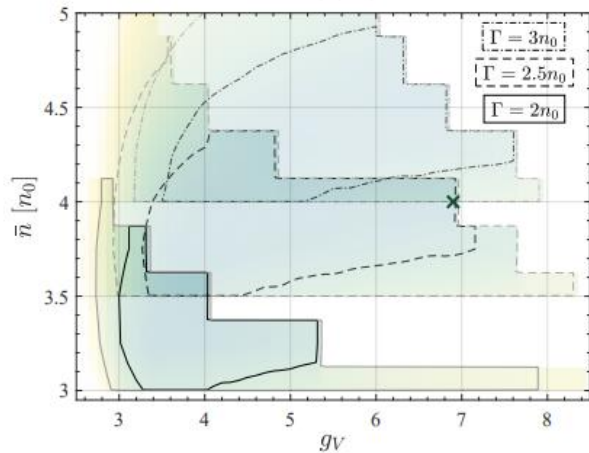
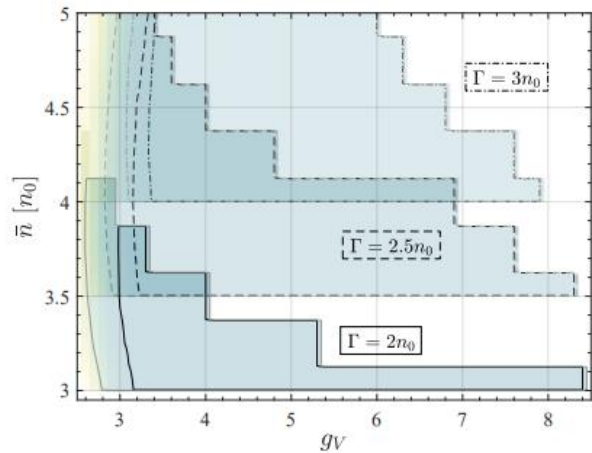
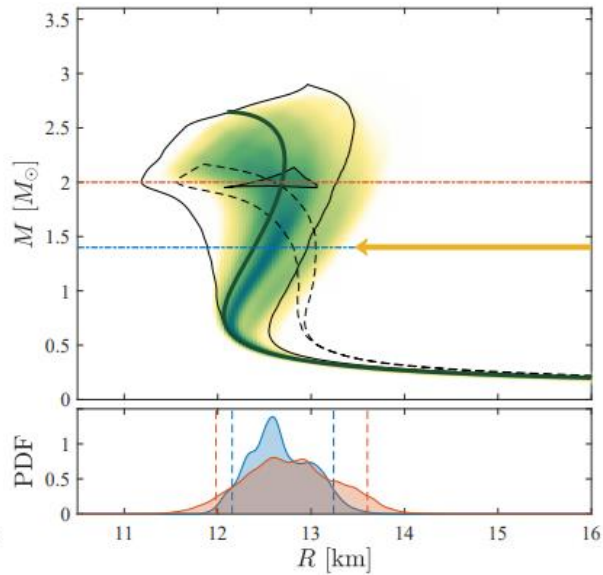
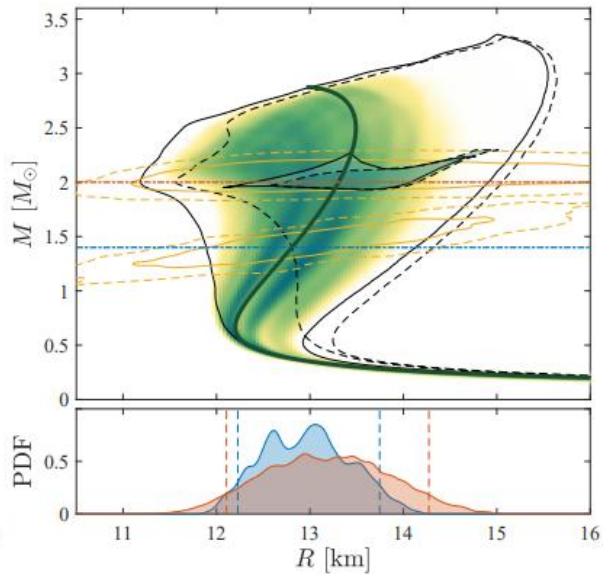
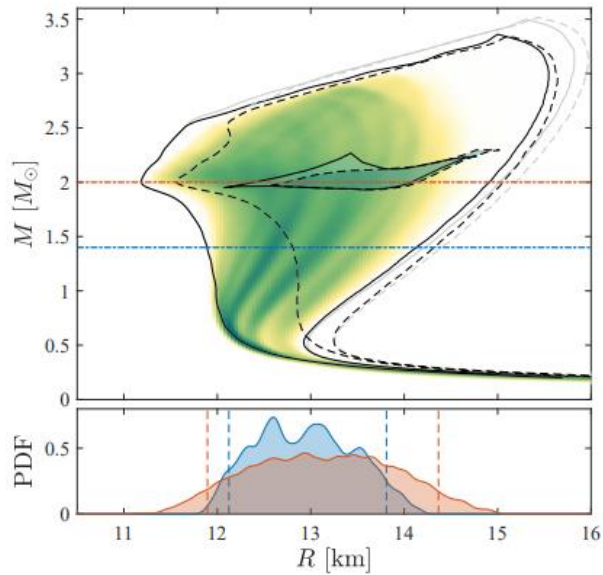
Two other measurements:

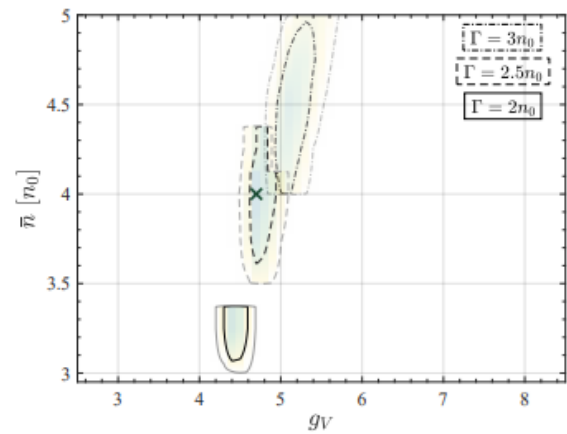
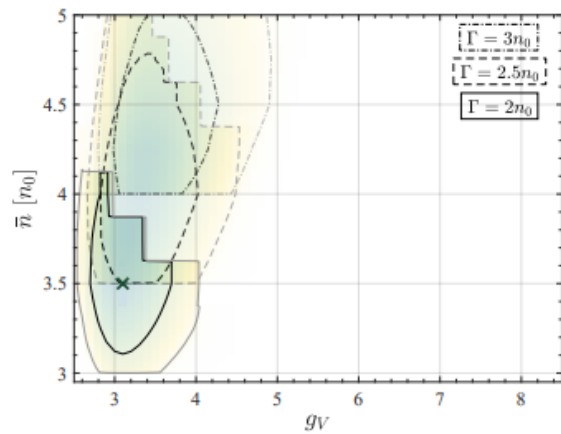
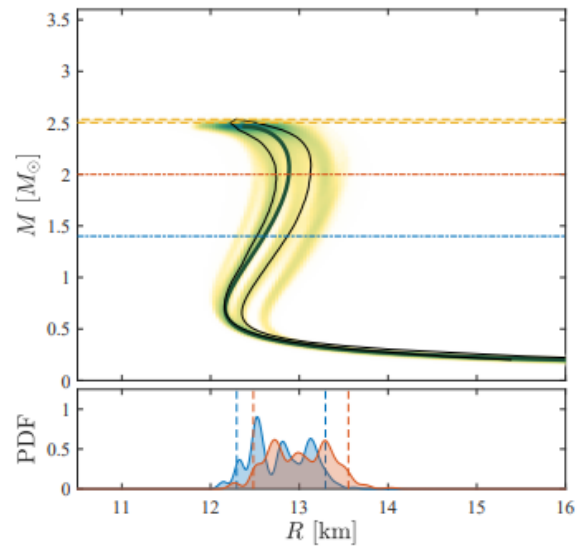
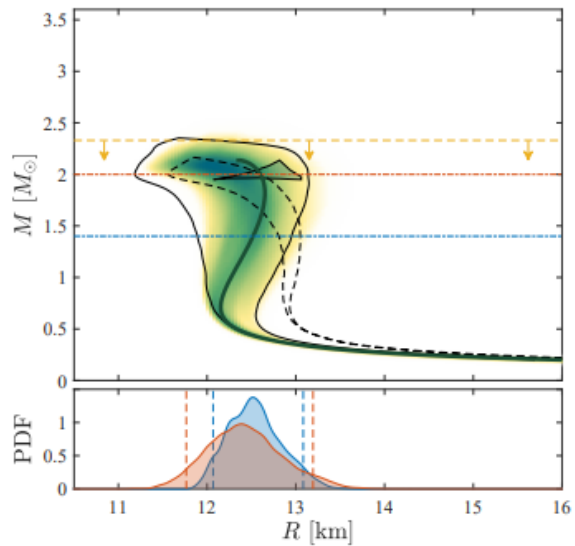
- 1) Projected semi-major axis of the companion star → mass ratio
  - a) White dwarf → optical measurements
  - b) Another pulsar
    - 2.1) Eclipses → observed edge-on
    - 2.2) Independent mass measurement of the companion: Shapiro-delay
    - 2.3) Relativistic effects: precession, gravitational radiation

$$f_{\text{ns}} = \left( \frac{2\pi}{P_{\text{orb}}} \right)^2 \frac{(a_{\text{ns}} \sin i)^3}{G} = \frac{(M_c \sin i)^3}{M_{\text{T}}^2}$$

$$q = \frac{M}{M_c} = \frac{(a_c \sin i)}{(a_{\text{ns}} \sin i)}$$









# Constraints on concatenation parameters

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