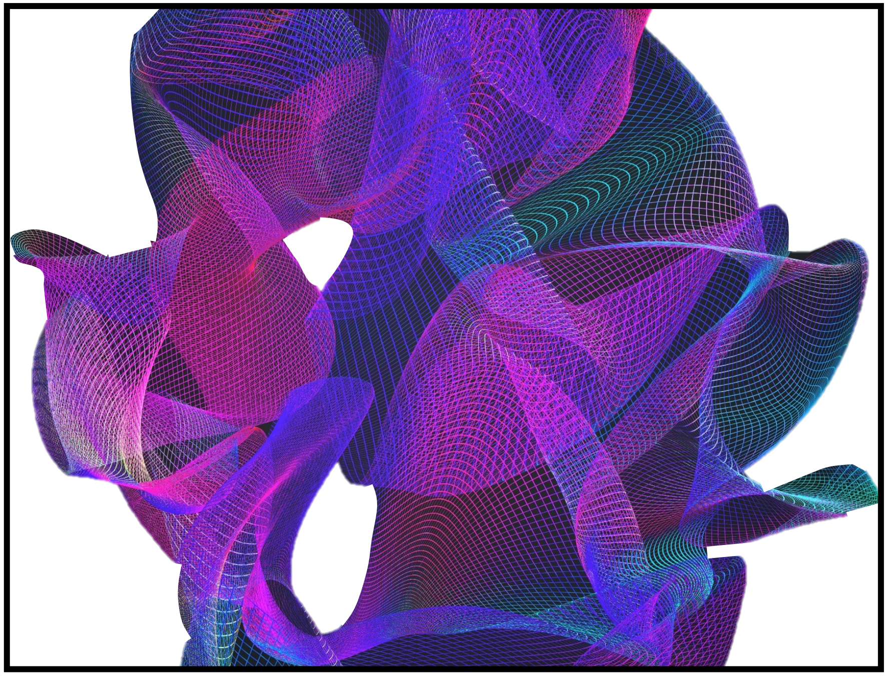
Supergeometry in Effective QFTs

Viola Gattus

18th June, International School of Subnuclear Physics 2024, Erice, Sicily



Based on V. Gattus and A. Pilaftsis, *Minimal supergeometric quantum field theories, Phys. Lett. B* **846** (2023) 138234 [2307.01126]

MANCHESTER 1824
The University of Manchester

ETTORE MAJORANA FOUNDATION AND CENTER FOR SCIENTIFIC CULTURE – EMFCSC

Motivation

Disclaimer: Supergeometry $\neq$ supersymmetry

Motivation

<u>Disclaimer</u>: Supergeometry ≠ supersymmetry

- A theory with fermions and bosons with no extra symmetry

Why supergeometry?

- Off-shell calculations are sensitive to choice of parametrisation

- Remove gauge-dependence in off-shell calculations

- Unique field-reparametrisation invariant expansion of geometric EFTs

[Alonso, Jenkins, Manohar (2016), Cohen, Craig, Sutherland (2021), Talbert (2023), Assi, Helset, Manohar, Pagès, Shen (2023) ...]

- Frame covariant description of cosmological inflation

[Burns, Karamitsos, Pilaftsis (2016), Falls, Herrero-Valea (2019), Finn, Karamitsos, Pilaftsis (2020) ..]

The Set-Up

- Field-space supermanifold of dimension (N|8M) in 4D spacetime
- Now fermions in the chart

[DeWitt (2012)]

$$\mathbf{\Phi} \;\equiv\; \{\Phi^\alpha\} \;=\; \left(\phi^A\,,\,\psi^X\,,\,\overline{\psi}^{Y,\mathsf{T}}\right)^{\mathsf{T}}$$

- Field reparameterization = diffeomorphism

$$\Phi^\alpha \;\to\; \widetilde{\Phi}^\alpha \;=\; \widetilde{\Phi}^\alpha(\mathbf{\Phi})$$

- Diffeomorphically - or frame invariant Lagrangian

$$\mathcal{L} \;=\; \frac{1}{2} g^{\mu\nu}\partial_\mu\Phi^\alpha\;{}_\alpha k_\beta(\mathbf{\Phi})\,\partial_\nu\Phi^\beta \;+\; \frac{i}{2}\,\zeta^\mu_\alpha(\mathbf{\Phi})\,\partial_\mu\Phi^\alpha \;-\; U(\mathbf{\Phi})$$

The Set-Up

- Field-space supermanifold of dimension (N|8M) in 4D spacetime
- Now fermions in the chart

[DeWitt (2012)]

$$\boldsymbol{\Phi} \;\equiv\; \{\Phi^\alpha\} \;=\; \left(\phi^A\,,\,\psi^X\,,\,\overline{\psi}^{Y,\mathsf{T}}\right)^{\mathsf{T}}$$

- Field reparameterization = diffeomorphism

$$\Phi^\alpha \;\to\; \widetilde{\Phi}^\alpha \;=\; \widetilde{\Phi}^\alpha(\boldsymbol{\Phi})$$

- Diffeomorphically - or frame invariant Lagrangian

$$\mathcal{L} \;=\; \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi^\alpha\,{}_\alpha k_\beta(\boldsymbol{\Phi})\,\partial_\nu\Phi^\beta \;+\; \frac{i}{2}\zeta^\mu_\alpha(\boldsymbol{\Phi})\,\partial_\mu\Phi^\alpha \;-\; U(\boldsymbol{\Phi})$$

$$\zeta^{\mu}_{\beta}{}^{\beta}\left(\overleftarrow{\Sigma}_\mu\right)_\alpha \;=\; \zeta_\alpha$$ where $$\overleftarrow{\Sigma}_\mu = \frac{1}{D}\begin{pmatrix}\frac{\overleftarrow{\partial}}{\partial\gamma^\mu} & 0\\ 0 & \Gamma_\mu\end{pmatrix}$$

The Set-Up (continued)

- Endow supermanifold with metric

$$_{\alpha}G_{\beta} = ({_{\alpha}G_{\beta}})^{\mathsf{sT}}$$

→

- supersymmetric rank-2 FS tensor
- ultralocal
- determined from action

The Set-Up (continued)

- Endow supermanifold with metric

$$ {}_\alpha G_\beta = ({}_\alpha G_\beta)^{\mathsf{sT}} $$

- supersymmetric rank-2 FS tensor
- ultralocal
- determined from action

- Global metric found from vielbeins and local metric

[Finn, Karamitsos, Pilaftsis (2021), VG, Finn, Karamitsos, Pilaftsis (2022)]

$$ {}_\alpha G_\beta = {}_\alpha e^a \, {}_a H_b \, {}^b e^{\mathsf{sT}}_\beta \; ; $$

$$ {}_a H_b \equiv \begin{pmatrix} \mathbf{1}_N & 0 & 0 \\ 0 & 0 & \mathbf{1}_{4M} \\ 0 & -\mathbf{1}_{4M} & 0 \end{pmatrix} $$

No-Go Theorem

- Flat field space can **always** be reparametrized into canonical Cartesian form

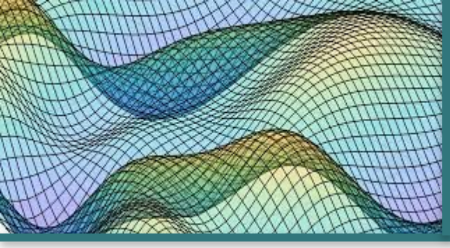
$$\mathcal{L} = -\frac{1}{2}\boldsymbol{h}(\phi)\,\overline{\boldsymbol{\psi}}\gamma^{\mu}\boldsymbol{\psi}(\partial_{\mu}\phi) + \frac{i}{2}\boldsymbol{g}(\phi)\left[\overline{\boldsymbol{\psi}}\gamma^{\mu}(\partial_{\mu}\boldsymbol{\psi}) - (\partial_{\mu}\overline{\boldsymbol{\psi}})\gamma^{\mu}\boldsymbol{\psi}\right]$$

No-Go Theorem

- Flat field space can **always** be reparametrized into canonical Cartesian form

$$\mathcal{L} = -\frac{1}{2}\boldsymbol{h}(\phi)\,\overline{\boldsymbol{\psi}}\gamma^\mu\boldsymbol{\psi}(\partial_\mu\phi) + \frac{i}{2}\boldsymbol{g}(\phi)\left[\overline{\boldsymbol{\psi}}\gamma^\mu(\partial_\mu\boldsymbol{\psi}) - (\partial_\mu\overline{\boldsymbol{\psi}})\gamma^\mu\boldsymbol{\psi}\right]$$

$$\overline{\boldsymbol{\psi}} = \{\overline{\psi}^X\} \qquad \boldsymbol{\psi} = \{\psi^X\} \qquad \boldsymbol{g}(\phi) = \{g_{XY}\}$$



No-Go Theorem

- Flat field space can **always** be reparametrized into canonical Cartesian form

$$\mathcal{L} = -\frac{1}{2}\boldsymbol{h}(\phi)\,\overline{\boldsymbol{\psi}}\gamma^{\mu}\boldsymbol{\psi}(\partial_{\mu}\phi) + \frac{i}{2}\boldsymbol{g}(\phi)\left[\overline{\boldsymbol{\psi}}\gamma^{\mu}(\partial_{\mu}\boldsymbol{\psi}) - (\partial_{\mu}\overline{\boldsymbol{\psi}})\gamma^{\mu}\boldsymbol{\psi}\right]$$

$$\overline{\boldsymbol{\psi}} = \{\overline{\psi}^{X}\} \qquad \boldsymbol{\psi} = \{\psi^{X}\} \qquad \boldsymbol{g}(\phi) = \{g_{XY}\}$$

- Reparameterise fields to make canonical

$$\boldsymbol{\psi} \longrightarrow \tilde{\boldsymbol{\psi}} = \boldsymbol{K}(\phi)^{-1}\boldsymbol{\psi}$$

No-Go Theorem

- Flat field space can **always** be reparametrized into canonical Cartesian form

$$\mathcal{L} = -\frac{1}{2}\boldsymbol{h}(\phi)\,\overline{\boldsymbol{\psi}}\gamma^\mu\boldsymbol{\psi}(\partial_\mu\phi) + \frac{i}{2}\boldsymbol{g}(\phi)\left[\overline{\boldsymbol{\psi}}\gamma^\mu(\partial_\mu\boldsymbol{\psi}) - (\partial_\mu\overline{\boldsymbol{\psi}})\gamma^\mu\boldsymbol{\psi}\right]$$

$$\overline{\boldsymbol{\psi}} = \{\overline{\psi}^X\} \qquad \boldsymbol{\psi} = \{\psi^X\} \qquad \boldsymbol{g}(\phi) = \{g_{XY}\}$$

- Reparameterise fields to make canonical

$$\boldsymbol{\psi} \longrightarrow \tilde{\boldsymbol{\psi}} = \boldsymbol{K}(\phi)^{-1}\boldsymbol{\psi}$$

$$\boldsymbol{K}(\phi) = \exp\left(-\frac{i}{2}\int_0^\phi \boldsymbol{g}^{-1}\boldsymbol{h}\,d\phi\right)$$

- ϕ acts as external parameter in the fermionic sector

No-Go Theorem (continued)

- Flatness confirmed by vanishing of Riemann tensor

$$
{}_{\alpha}G_{\beta} \;=\; \begin{pmatrix} k - \frac{1}{2}\overline{\boldsymbol{\psi}}\,(\boldsymbol{g}' - i\boldsymbol{h})\,\boldsymbol{g}^{-1}\,(\boldsymbol{g}' + i\boldsymbol{h})\,\boldsymbol{\psi} & -\frac{1}{2}\overline{\boldsymbol{\psi}}\,(\boldsymbol{g}' - i\boldsymbol{h}) & \frac{1}{2}\boldsymbol{\psi}^{\mathsf{T}}\left(\boldsymbol{g}'^{\mathsf{T}} + i\boldsymbol{h}^{\mathsf{T}}\right) \\ \frac{1}{2}\left(\boldsymbol{g}'^{\mathsf{T}} - i\boldsymbol{h}^{\mathsf{T}}\right)\overline{\boldsymbol{\psi}}^{\mathsf{T}} & 0 & \boldsymbol{g}^{\mathsf{T}}1_4 \\ -\frac{1}{2}\,(\boldsymbol{g}' + i\boldsymbol{h})\,\boldsymbol{\psi} & -\boldsymbol{g}1_4 & 0 \end{pmatrix}
$$

$$
\dashrightarrow \quad R^{\alpha}{}_{\beta\gamma\delta} \;=\; 0
$$

[VG, Pilaftsis (2023)]

No-Go Theorem (continued)

- Flatness confirmed by vanishing of Riemann tensor

$$
{}_{\alpha}G_{\beta} \;=\; \begin{pmatrix} k - \frac{1}{2}\overline{\boldsymbol{\psi}}\left(\boldsymbol{g}' - i\boldsymbol{h}\right)\boldsymbol{g}^{-1}\left(\boldsymbol{g}' + i\boldsymbol{h}\right)\boldsymbol{\psi} & -\frac{1}{2}\overline{\boldsymbol{\psi}}\left(\boldsymbol{g}' - i\boldsymbol{h}\right) & \frac{1}{2}\boldsymbol{\psi}^{\mathsf{T}}\left(\boldsymbol{g}'^{\mathsf{T}} + i\boldsymbol{h}^{\mathsf{T}}\right) \\ \frac{1}{2}\left(\boldsymbol{g}'^{\mathsf{T}} - i\boldsymbol{h}^{\mathsf{T}}\right)\overline{\boldsymbol{\psi}}^{\mathsf{T}} & 0 & \boldsymbol{g}^{\mathsf{T}}1_4 \\ -\frac{1}{2}\left(\boldsymbol{g}' + i\boldsymbol{h}\right)\boldsymbol{\psi} & -\boldsymbol{g}1_4 & 0 \end{pmatrix}
$$

$$
\dashrightarrow \qquad R^{\alpha}{}_{\beta\gamma\delta} \;=\; 0
$$

[VG, Pilaftsis (2023)]

<u>Take home message:</u>

Non-zero fermionic curvature effects cannot be generated if ζ^{μ}_{α} depends **linearly** on ψ and $\overline{\psi}$

Model I

- A 2D factorizable model

$$\mathcal{L}_{\mathrm{I}} = \frac{1}{2}k\,(\partial_\mu\phi)\,(\partial^\mu\phi) + \frac{i}{2}\left(g_0 + g_1\overline{\psi}\psi\right)\left[\overline{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^\mu\psi\right]$$

Model I

- A 2D factorizable model

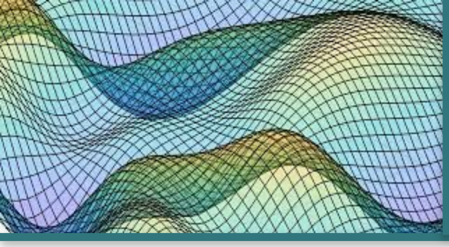
$$\mathcal{L}_{\mathrm{I}} = \frac{1}{2}k\,(\partial_\mu\phi)\,(\partial^\mu\phi) + \frac{i}{2}\left(g_0 + g_1\overline{\psi}\psi\right)\left[\overline{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^\mu\psi\right]$$

$$\zeta^\mu_\alpha = \zeta_\beta{}^\beta(\Gamma^\mu)_\alpha \quad \text{where} \quad \Gamma^\mu = \begin{pmatrix} \gamma^\mu & 0 \\ 0 & (\gamma^\mu)^\mathsf{T} \end{pmatrix}$$

Model I

- A 2D factorizable model

$$\mathcal{L}_{\mathrm{I}} = \frac{1}{2}k\,(\partial_\mu \phi)\,(\partial^\mu \phi) + \frac{i}{2}\left(g_0 + g_1 \overline{\psi}\psi\right)\left[\overline{\psi}\gamma^\mu(\partial_\mu \psi) - (\partial_\mu \overline{\psi})\gamma^\mu \psi\right]$$



Model I

- A 2D factorizable model

$$\mathcal{L}_{\mathrm{I}} = \frac{1}{2}k\,(\partial_\mu \phi)\,(\partial^\mu \phi) + \frac{i}{2}\left(g_0 + g_1\overline{\psi}\psi\right)\left[\overline{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^\mu\psi\right]$$

- FS metric

$$\boldsymbol{G} = \begin{pmatrix} k + b^{\mathsf{T}}(d^{-1})^{\mathsf{T}}a^{\mathsf{T}} - a\,d^{-1}\,b & -a & b^{\mathsf{T}} \\ a^{\mathsf{T}} & 0 & d^{\mathsf{T}} \\ -b & -d & 0 \end{pmatrix}$$

$$a = \frac{1}{2}\overline{\psi}\left(g_0' + g_1'\overline{\psi}\psi\right)$$

$$b = \frac{1}{2}\left(g_0' + g_1'\overline{\psi}\psi\right)\psi$$

$$d = \left(g_0 + g_1\overline{\psi}\psi\right)1_2 + g_1\psi\overline{\psi}$$

Model I

- A 2D factorizable model

$$\mathcal{L}_{\mathrm{I}} = \frac{1}{2}k\,(\partial_\mu\phi)\,(\partial^\mu\phi) + \frac{i}{2}\left(g_0 + g_1\overline{\psi}\psi\right)\left[\overline{\psi}\gamma^\mu(\partial_\mu\psi) - (\partial_\mu\overline{\psi})\gamma^\mu\psi\right]$$

- FS metric

$$\boldsymbol{G} = \begin{pmatrix} k + b^{\mathsf{T}}(d^{-1})^{\mathsf{T}}a^{\mathsf{T}} - a\,d^{-1}\,b & -a & b^{\mathsf{T}} \\ a^{\mathsf{T}} & 0 & d^{\mathsf{T}} \\ -b & -d & 0 \end{pmatrix}$$

$$a = \frac{1}{2}\overline{\psi}\left(g_0' + g_1'\overline{\psi}\psi\right)$$

$$b = \frac{1}{2}\left(g_0' + g_1'\overline{\psi}\psi\right)\psi$$

$$d = \left(g_0 + g_1\overline{\psi}\psi\right)1_2 + g_1\psi\overline{\psi}$$

- Ricci scalar is fermion dependent

$$R = \frac{4g_1}{g_0^2} + \left(\frac{2g_1g_0'g_1'}{g_0^3k} - \frac{2g_1^2{g_0'}^2}{g_0^4k} - \frac{{g_1'}^2}{2g_0^2k}\right)(\overline{\psi}\psi)^2$$

Supervertices: the full picture

The goal: off-shell vertices in the non-static limit for scalar-fermion theories

- Notation

$$\partial_\mu \Phi^{\alpha}{}_{;\beta} = \partial_\mu^{(\alpha)} \delta(x_\alpha - x_\beta)\, \delta^{\alpha}{}_{\beta} + \Gamma^{\alpha}{}_{\beta\rho}\, \partial_\mu \Phi^{\rho}\, \delta(x_\alpha - x_\beta) := (D_\mu)^{\alpha}{}_{\beta}$$

$$\{\alpha\beta\} = \alpha\beta + (-1)^{\alpha\beta}\beta\alpha$$

Supervertices: the full picture

The goal: off-shell vertices in the non-static limit for scalar-fermion theories

- Notation

$$\partial_\mu \Phi^\alpha{}_{;\beta} = \partial_\mu^{(\alpha)} \delta(x_\alpha - x_\beta)\, \delta^\alpha{}_\beta + \Gamma^\alpha{}_{\beta\rho}\, \partial_\mu \Phi^\rho\, \delta(x_\alpha - x_\beta) := (D_\mu)^\alpha{}_\beta$$

$$\{\alpha\beta\} = \alpha\beta + (-1)^{\alpha\beta} \beta\alpha$$

[VG, Pilaftsis, 2023/2024]

- Complete covariant inverse superpropagator

$$\begin{aligned} S_{;\hat{\alpha}\hat{\beta}} = \partial_\mu \Phi^\rho \Big(& {}_\rho k_\gamma\, R^\gamma{}_{\alpha\beta\delta}\, \partial^\mu \Phi^\delta + (-1)^{\alpha\gamma}\, {}_\rho k_{\gamma;\{\alpha}\, (D^\mu)^\gamma{}_{\beta\}} \\ & + \frac{1}{2}(-1)^{\gamma(\alpha+\beta)}\, {}_\rho k_{\gamma;\alpha\beta}\, \partial^\mu \Phi^\gamma \Big) + (-1)^{\alpha\rho} (D_\mu)^\rho{}_\alpha\, {}_\rho k_\gamma\, (D^\mu)^\gamma{}_\beta \\ & + i(-1)^\alpha \left({}_\alpha\lambda^\mu_\rho (D_\mu)^\rho{}_\beta + (-1)^{\beta\rho}\, {}_\alpha\lambda^\mu_{\rho;\beta}\, \partial_\mu \Phi^\rho \right) \end{aligned}$$

Supervertices: the full picture

The goal: off-shell vertices in the non-static limit for scalar-fermion theories

- Notation

$$\partial_\mu \Phi^\alpha{}_{;\beta} = \partial_\mu^{(\alpha)} \delta(x_\alpha - x_\beta)\, \delta^\alpha_\beta + \Gamma^\alpha{}_{\beta\rho}\, \partial_\mu \Phi^\rho\, \delta(x_\alpha - x_\beta) := (D_\mu)^\alpha{}_\beta$$

$$\{\alpha\beta\} = \alpha\beta + (-1)^{\alpha\beta}\beta\alpha$$

[VG, Pilaftsis, 2023/2024]

- Complete covariant inverse superpropagator

bosonic sector

$$S_{;\hat\alpha\hat\beta} = \partial_\mu \Phi^\rho \Big({}_\rho k_\gamma\, R^\gamma{}_{\alpha\beta\delta}\, \partial^\mu \Phi^\delta + (-1)^{\alpha\gamma}\, {}_\rho k_{\gamma;\{\alpha}\, (D^\mu)^\gamma{}_{\beta\}}$$
$$+ \frac{1}{2}(-1)^{\gamma(\alpha+\beta)}\, {}_\rho k_{\gamma;\alpha\beta}\, \partial^\mu \Phi^\gamma \Big) + (-1)^{\alpha\rho} (D_\mu)^\rho{}_\alpha\, {}_\rho k_\gamma\, (D^\mu)^\gamma{}_\beta$$
$$+ i(-1)^\alpha \left({}_\alpha\lambda^\mu_\rho (D_\mu)^\rho{}_\beta + (-1)^{\beta\rho}\, {}_\alpha\lambda^\mu_{\rho;\beta}\, \partial_\mu \Phi^\rho \right)$$

Supervertices: the full picture

The goal: off-shell vertices in the non-static limit for scalar-fermion theories

- Notation

$$\partial_\mu \Phi^\alpha{}_{;\beta} = \partial_\mu^{(\alpha)} \delta(x_\alpha - x_\beta)\, \delta^\alpha{}_\beta + \Gamma^\alpha{}_{\beta\rho}\, \partial_\mu \Phi^\rho\, \delta(x_\alpha - x_\beta) := (D_\mu)^\alpha{}_\beta$$

$$\{\alpha\beta\} = \alpha\beta + (-1)^{\alpha\beta} \beta\alpha$$

[VG, Pilaftsis, 2023/2024]

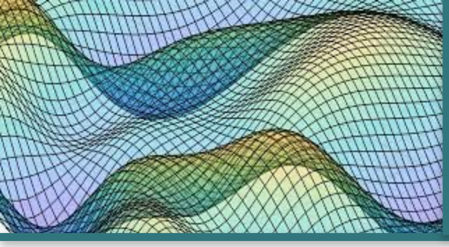
- Complete covariant inverse superpropagator

$$S_{;\hat{\alpha}\hat{\beta}} = \partial_\mu \Phi^\rho \Big({}_\rho k_\gamma\, R^\gamma{}_{\alpha\beta\delta}\, \partial^\mu \Phi^\delta + (-1)^{\alpha\gamma}\, {}_\rho k_{\gamma;\{\alpha}\, (D^\mu)^\gamma{}_{\beta\}}$$

$$+ \frac{1}{2}(-1)^{\gamma(\alpha+\beta)}\, {}_\rho k_{\gamma;\alpha\beta}\, \partial^\mu \Phi^\gamma \Big) + (-1)^{\alpha\rho} (D_\mu)^\rho{}_\alpha\, {}_\rho k_\gamma\, (D^\mu)^\gamma{}_\beta$$

fermionic sector

$$+ i(-1)^\alpha \Big({}_\alpha \lambda^\mu_\rho (D_\mu)^\rho{}_\beta + (-1)^{\beta\rho}\, {}_\alpha \lambda^\mu_{\rho;\beta}\, \partial_\mu \Phi^\rho \Big)$$



Covariant effective action

[Vilkovisky (1984), DeWitt (1985), Finn, Karamitsos, Pilaftsis, 2022]

- Implicit equation for the effective action using VDW

$$\exp\left(\frac{i}{\hbar}\Gamma[\mathbf{\Phi}_0]\right) = \int \sqrt{|\,\mathrm{sdet}\, G|}\,[\mathcal{D}\mathbf{\Phi}]\,\exp\left(\frac{i}{\hbar}S\,[\mathbf{\Phi}] + \frac{i}{\hbar}\int d^4x\sqrt{-g}\,\Gamma[\mathbf{\Phi}_0]_{,\alpha}\,\Sigma^{\alpha}\,[\mathbf{\Phi}_0,\mathbf{\Phi}]\right)$$

Covariant effective action

[Vilkovisky (1984), DeWitt (1985), Finn, Karamitsos, Pilaftsis, 2022]

- Implicit equation for the effective action using VDW

$$\exp\left(\frac{i}{\hbar}\Gamma[\mathbf{\Phi}_0]\right) = \int \sqrt{|\,\mathrm{sdet}\, G|}\,[\mathcal{D}\mathbf{\Phi}]\exp\left(\frac{i}{\hbar}S\left[\mathbf{\Phi}\right] + \frac{i}{\hbar}\int d^4x\sqrt{-g}\,\Gamma[\mathbf{\Phi}_0]_{,\alpha}\,\Sigma^{\alpha}\left[\mathbf{\Phi}_0,\mathbf{\Phi}\right]\right)$$

- One and two loop expressions

$$\Gamma^{(1)}[\mathbf{\Phi}_0] = \frac{i}{2}\ln \mathrm{sdet}\ {}^{\hat{\alpha}}S_{\hat{\beta}} = \frac{i}{2}\,\mathrm{str}\ln\ {}^{\hat{\alpha}}S_{\hat{\beta}}$$

Covariant effective action

[Vilkovisky (1984), DeWitt (1985), Finn, Karamitsos, Pilaftsis, 2022]

- Implicit equation for the effective action using VDW

$$\exp\left(\frac{i}{\hbar}\Gamma[\mathbf{\Phi}_0]\right) = \int \sqrt{|\,\mathrm{sdet}\, G|}\,[\mathcal{D}\mathbf{\Phi}]\exp\left(\frac{i}{\hbar}S\,[\mathbf{\Phi}] + \frac{i}{\hbar}\int d^4x\sqrt{-g}\,\Gamma[\mathbf{\Phi}_0]_{,\alpha}\,\Sigma^{\alpha}\,[\mathbf{\Phi}_0,\mathbf{\Phi}]\right)$$

- One and two loop expressions

$$\Gamma^{(1)}[\mathbf{\Phi}_0] = \frac{i}{2}\ln\mathrm{sdet}\ {}^{\hat{\alpha}}S_{\hat{\beta}} = \frac{i}{2}\mathrm{str}\ln\ {}^{\hat{\alpha}}S_{\hat{\beta}}$$

$${}^{\hat{\alpha}}S_{\hat{\beta}} = {}^{\hat{\alpha}}\overrightarrow{\nabla}S\overleftarrow{\nabla}_{\hat{\beta}}$$

Covariant effective action

[Vilkovisky (1984), DeWitt (1985), Finn, Karamitsos, Pilaftsis, 2022]

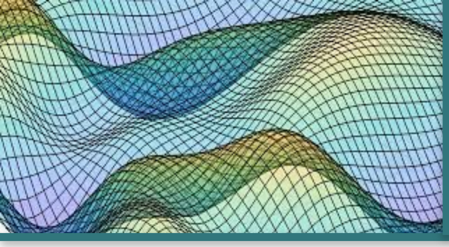
- Implicit equation for the effective action using VDW

$$\exp\left(\frac{i}{\hbar}\Gamma[\mathbf{\Phi}_0]\right) = \int \sqrt{|\,\mathrm{sdet}\, G|}\,[\mathcal{D}\mathbf{\Phi}]\exp\left(\frac{i}{\hbar}S\,[\mathbf{\Phi}] + \frac{i}{\hbar}\int d^4x\sqrt{-g}\,\Gamma[\mathbf{\Phi}_0]_{,\alpha}\,\Sigma^{\alpha}\,[\mathbf{\Phi}_0,\mathbf{\Phi}]\right)$$

- One and two loop expressions

$$\Gamma^{(1)}[\mathbf{\Phi}_0] = \frac{i}{2}\ln\mathrm{sdet}\ {}^{\hat\alpha}S_{\hat\beta} = \frac{i}{2}\,\mathrm{str}\ln\ {}^{\hat\alpha}S_{\hat\beta}$$

$$\Gamma^{(2)}[\mathbf{\Phi}_0] = -\frac{1}{8}S_{\{\hat\alpha\hat\beta\hat\gamma\hat\delta\}}\Delta^{\delta\hat\gamma}\Delta^{\hat\beta\hat\alpha} + \frac{1}{12}(-1)^{\hat\beta\hat\gamma+\hat\epsilon(\hat\beta+\hat\delta)}\,S_{\{\hat\epsilon\hat\gamma\hat\alpha\}}\Delta^{\hat\alpha\hat\beta}\Delta^{\hat\gamma\delta}\Delta^{\hat\epsilon\hat\zeta}{}_{\{\hat\zeta\hat\delta\hat\beta\}}S$$



Covariant effective action

[Vilkovisky (1984), DeWitt (1985), Finn, Karamitsos, Pilaftsis, 2022]

- Implicit equation for the effective action using VDW

$$\exp\left(\frac{i}{\hbar}\Gamma[\mathbf{\Phi}_0]\right) = \int \sqrt{|\operatorname{sdet} G|}\,[\mathcal{D}\mathbf{\Phi}]\exp\left(\frac{i}{\hbar}S\,[\mathbf{\Phi}] + \frac{i}{\hbar}\int d^4x\sqrt{-g}\,\Gamma[\mathbf{\Phi}_0]_{,\alpha}\,\Sigma^{\alpha}\,[\mathbf{\Phi}_0,\mathbf{\Phi}]\right)$$

- One and two loop expressions

$$\Gamma^{(1)}[\mathbf{\Phi}_0] = \frac{i}{2}\ln\operatorname{sdet}\,{}^{\hat\alpha}S_{\hat\beta} = \frac{i}{2}\operatorname{str}\ln\,{}^{\hat\alpha}S_{\hat\beta}$$

$$\Gamma^{(2)}[\mathbf{\Phi}_0] = -\frac{1}{8}S_{\{\hat\alpha\hat\beta\hat\gamma\hat\delta\}}\Delta^{\delta\hat\gamma}\Delta^{\hat\beta\hat\alpha} + \frac{1}{12}(-1)^{\hat\beta\hat\gamma+\hat\epsilon(\hat\beta+\hat\delta)}\,S_{\{\hat\epsilon\hat\gamma\hat\alpha\}}\Delta^{\hat\alpha\hat\beta}\Delta^{\hat\gamma\delta}\Delta^{\hat\epsilon\hat\zeta}{}_{\{\hat\zeta\hat\delta\hat\beta\}}S$$

$$\Delta^{\hat\alpha\hat\gamma}{}_{\hat\gamma}S_{\hat\beta} = {}^{\hat\alpha}\delta_{\hat\beta}$$

Explicit computation of $\Gamma^{(1)}$

- Flat FS example

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{i}{2}\partial_\mu\bar{\psi}\gamma^\mu\psi - Y(\phi)\bar{\psi}\psi - V(\phi)$$

- One-loop covariant effective action

$$\Gamma[\Phi] = \frac{i}{2}\,\mathrm{Tr}\ln\left\{\Box + V''(\phi) - \bar{\psi}\left[2Y'(\phi)(-i\partial\!\!\!/ + Y(\phi))^{-1}Y'(\phi) - Y''(\phi)\right]\psi\right\}$$
$$- i\,\mathrm{Tr}\ln\Big(-i\partial\!\!\!/ + Y(\phi)\Big)$$

Explicit computation of $\Gamma^{(1)}$

- Flat FS example

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{i}{2}\partial_\mu\bar{\psi}\gamma^\mu\psi - Y(\phi)\bar{\psi}\psi - V(\phi)$$

- One-loop covariant effective action

$$\Gamma[\Phi] = \frac{i}{2}\,\mathrm{Tr}\ln\left\{\Box + V''(\phi) - \bar{\psi}\left[2Y'(\phi)(-i\partial\!\!\!/ + Y(\phi))^{-1}Y'(\phi) - Y''(\phi)\right]\psi\right\}$$
$$- i\,\mathrm{Tr}\ln\left(-i\partial\!\!\!/ + Y(\phi)\right)$$

Schwinger's proper time representation
+
Zassenhaus formula

Explicit computation of $\Gamma^{(1)}$

- Flat FS example

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{i}{2}\partial_\mu\bar{\psi}\gamma^\mu\psi - Y(\phi)\bar{\psi}\psi - V(\phi)$$

- One-loop covariant effective action

$$\Gamma[\Phi] = \frac{i}{2}\,\mathrm{Tr}\ln\left\{\Box + V''(\phi) - \bar{\psi}\left[2Y'(\phi)(-i\partial\!\!\!/ + Y(\phi))^{-1}Y'(\phi) - Y''(\phi)\right]\psi\right\}$$
$$- i\,\mathrm{Tr}\ln\left(-i\partial\!\!\!/ + Y(\phi)\right)$$

- Compute second term explicitly

$$\propto \frac{i}{2}\,\mathrm{Tr}\int_0^\infty \frac{dt}{t} e^{-t\left[\Box + (m_f + h\phi)^2 - ih\partial\!\!\!/\phi\right]}$$

Explicit computation of $\Gamma^{(1)}$

- Flat FS example

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{i}{2}\bar{\psi}\gamma^\mu\partial_\mu\psi - \frac{i}{2}\partial_\mu\bar{\psi}\gamma^\mu\psi - Y(\phi)\bar{\psi}\psi - V(\phi)$$

- One-loop covariant effective action

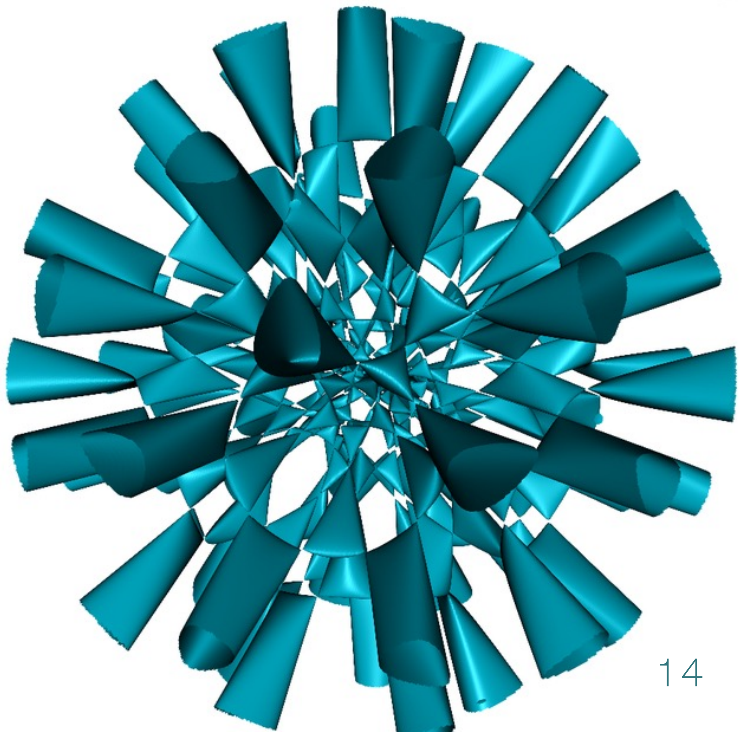
$$\Gamma[\Phi] = \frac{i}{2}\,\mathrm{Tr}\ln\left\{\Box + V''(\phi) - \bar{\psi}\left[2Y'(\phi)(-i\partial\!\!\!/ + Y(\phi))^{-1}Y'(\phi) - Y''(\phi)\right]\psi\right\}$$
$$- i\,\mathrm{Tr}\ln\left(-i\partial\!\!\!/ + Y(\phi)\right)$$

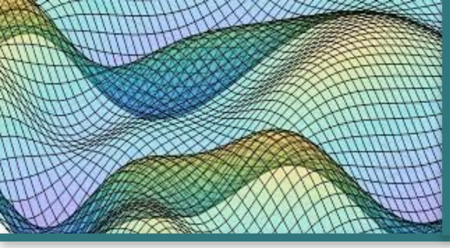
- Extract UV poles

$$= \frac{i}{2}\int d^dx \frac{d}{(4\pi)^{d/2}}\left\{-\left(h^2\phi^2 + 2hm_f\phi\right)\Gamma\left(1-\frac{d}{2}\right)\right.$$
$$\left. + \frac{1}{2}\left(\left(h^2\phi^2 + 2hm_f\phi\right)^2 + h^2\left(\partial_\mu\phi\right)^2\right)\Gamma\left(2-\frac{d}{2}\right) + \text{finite}\right\}$$

Summary and Outlook

- Fermionic curvature arises from non-linearity
- Unlike supergravity, curvature not real-valued
- Derived generalised expressions for covariant scalar-fermion vertices



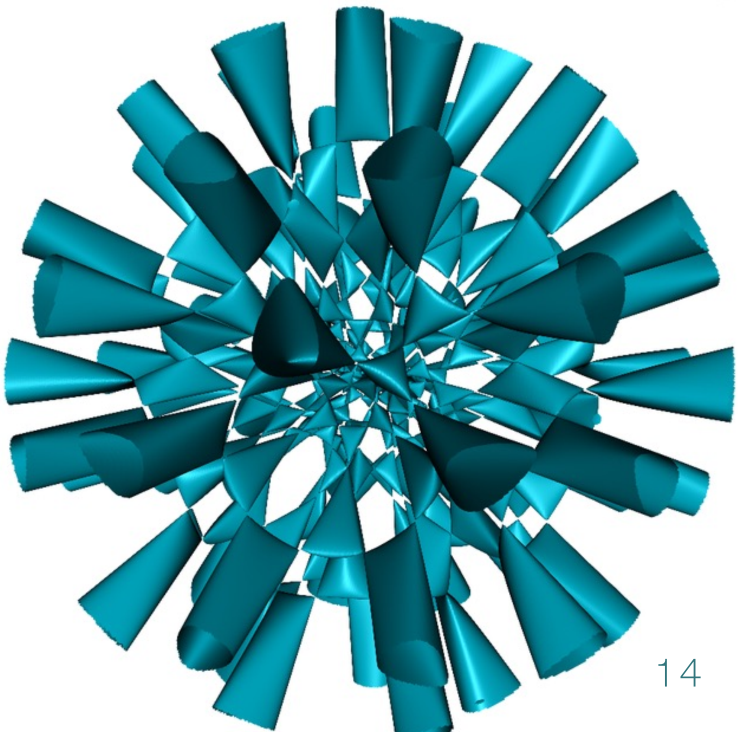


Summary and Outlook

- ❑ Fermionic curvature arises from non-linearity
- ❑ Unlike supergravity, curvature not real-valued
- ❑ Derived generalised expressions for covariant scalar-fermion vertices

What next?

- ❑ Compute higher loop effective actions
- ❑ Add symmetries
- ❑ Compute amplitudes



Thank you!