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based on works:

[1] PRL 129(15): 151601, (2022)
[2] PLB 840, 137839, (2023)
[3] PRD 108, 12, L121701 (2023)
[4] PRD 109, 10, 105001 (2024)

Hydrodynamic Manifestations of Gravitational Chiral Anomaly

The 2024 Course of the «Ettore Majorana» International School of Subnuclear Physics,
Erice, Sicily, Italy,
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Contents

Part 1

Introduction

Gravity in flat spacetime: Cheshire cat grin

“Well! I've often seen a cat without a grin,' thought Alice 'but a grin without a cat! It's the most curious thing i ever saw in my life!”

— Lewis Carroll, Alice in Wonderland

Gravity in flat spacetime: Cheshire cat grin

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— Lewis Carroll, Alice in Wonderland

CVE and CME – new anomalous transport

Consistency with **quantum anomaly** modifies hydrodynamic equations

[V. I. Zakharov, Lect. Notes Phys.871,295(2013)]

[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]

Quantum chiral anomaly (gauge part)

$$\langle \partial_\mu \hat{j}^\mu_A \rangle = -\frac{C}{8}\varepsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$$

Entropy current satisfies **second law** of thermodynamics $\partial_\mu s^\mu \geqslant 0$

Chiral magnetic effect (CME)

CME: $j^\mu = C\mu_5 B^\mu$

Current flows along the **magnetic field**

Chiral vortical effect (CVE)

CVE: $j^\mu_A = C\mu^2\omega^\mu$

$$\omega^\mu = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta$$

Current flows along the **vorticity**

Derivation **without entropy** current and generalization to the **second order** in gradients:

[Shi-Zheng Yang, Jian-Hua Gao, Zuo-Tang Liang, Symmetry 14 (2022) 5, 948]

[M. Buzzegoli, Lect. Notes Phys. 987, 53-93 (2021)]

Use **global equilibrium**

Modern development and the problem

What about the **gravitational chiral anomaly**?

- The gravitational chiral anomaly (unlike gauge part) grows **rapidly** with **spin**:

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_S = \frac{(S - 2S^3)}{96\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, in First School on Supergravity (1982) arXiv:1201.0386]

[S. M. Christensen, M. J. Duff, Nucl. Phys. B 154, 301–342 (1979)]

- How does the **gravitational** chiral anomaly manifest itself in **hydrodynamics**?
- Is it possible to see the **cubic factor** $S - 2S^3$ in hydrodynamics?

General derivation

Decomposition of the tensors

- **Components of the thermal vorticity tensor** **6** components

[M. Buzzegoli, E. Grossi, F. Becattini, JHEP 10 (2017) 091]

$$\varpi_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta} w^\alpha u^\beta + \alpha_\mu u_\nu - \alpha_\nu u_\mu$$

Similar to the expansion for the electromagnetic field

- We also decompose the **Riemann tensor** into the **components:**

$$\begin{aligned} R_{\mu\nu\alpha\beta} = & \ u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\ & + \epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda{}_\beta - u_\beta B^\lambda{}_\alpha) \\ & + \epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda{}_\nu - u_\nu B^\lambda{}_\mu) \\ & + \epsilon_{\mu\nu\lambda\rho} \epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta} \end{aligned}$$

20 components

Coincide with **3d** tensors in the fluid rest frame:

[L. D. Landau and E. M. Lifschits, The Classical Theory of Fields, Vol. 2, 1975]

[A. Z. Petrov, 1950]

- We consider **Ricci-flat** spaces $R_{\mu\nu} = 0$

Gradient expansion in the curved spacetime

The **gravitational chiral anomaly** has the **4th order** in gradients – it is to be related to the **3rd order** terms in gradient expansion of the axial **current**.

Hydrodynamic expansion of the axial current up to the **3rd order** in gradients:

Survive in flat spacetime

$$j_\mu^{A(3)} = \xi_1(T) w^2 w_\mu + \xi_2(T)\alpha^2 w_\mu + \xi_3(T)(\alpha w) w_\mu + \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu}\alpha^\nu$$

arbitrary coefficients

"gravitational" currents

See also gradient expansion for the fluid in the gravitational field, e.g.:

[P. Romatschke, Class. Quant. Grav. 27, 025006 (2010)]

[S. M. Diles, L. A. H. Mamani, A. S. Miranda, V. T. Zanchin, JHEP 2020, 1–40 (2020)]

ANOMALY MATCHING: PRINCIPLE

Following [D.T. Son, P. Surowka, PRL, 103 (2009) 191601]
- it would be necessary to construct the **entropy current**.

However in [Shi-Zheng Yang, Jian-Hua Gao, and Zuo-Tang Liang, Symmetry 14, 948 (2022)]

it is shown that it is possible to use the **global equilibrium** condition

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0$$

After that it is enough to consider ***only*** the equation for the current.

- Good for gravity, which is complicated in general case!

We use only:

$$\nabla_\mu j^\mu_A = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

Substitute the gradient expansion:

$$\nabla^\mu \Big(\xi_1(T) w^2 w_\mu + \xi_2(T) \alpha^2 w_\mu + \xi_3(T)(\alpha w) w_\mu + \xi_4(T) A_{\mu\nu} w^\nu + \xi_5(T) B_{\mu\nu} \alpha^\nu \Big) = 32\mathcal{N} A_{\mu\nu} B^{\mu\nu}$$

Anomaly matching: system of equations

This **system** of linear **differential** equations has the form:

$$
\begin{aligned}
-3T\xi_1 + T^2\xi_1' + 2T\xi_3 &= 0 \\
-3T\xi_2 + T^2\xi_2' - T\xi_3 + T^2\xi_3' &= 0 \\
T^2\xi_4' + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3 &= 0 \\
-2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5 &= 0 \\
T^2\xi_5' - T\xi_5 - T^{-1}\xi_3 &= 0 \\
-T^{-1}\xi_4 + T^{-1}\xi_5 - 32\mathscr{N} &= 0
\end{aligned}
$$

Can be solved!

Includes the factor from the **gravitational chiral anomaly**

Solution

Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains** a contribution to the axial current induced by the **gravitational chiral anomaly:**

Zero gravitational field

Limit $R_{\mu\nu\alpha\beta} = 0$

$$j_\mu^A = \lambda_1 (\omega_\nu \omega^\nu) \omega_\mu + \lambda_2 (a_\nu a^\nu) w_\mu$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Nonzero gravitational field

$$R_{\mu\nu\alpha\beta} \neq 0$$

$$\nabla_\mu j_A^\mu = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

- A new type of anomalous transport – the **Kinematical Vortical Effect (KVE)**. Does not explicitly depend on temperature and density → determined only by the **kinematics** of the flow.

Direct verification: spin 1/2

Transport coefficients and anomaly: spin 1/2

- In [GP, O.V. Teryaev, and V.I. Zakharov, JHEP, 02:146, 2019], [V. E. Ambrus, JHEP, 08:016, 2020], [A. Palermo, et al. JHEP 10 (2021) 077] and for ω^3 in [A. Vilenkin, Phys. Rev., D20:1807-1812, 1979] the following expression was obtained:

KVE

$$j_\mu^A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2}\right)\omega_\mu$$

- Comparing it with the well-known anomaly [L. Alvarez-Gaume, E. Witten, Nucl. Phys., B234:269, 1984]:

$$\nabla_\mu j_A^\mu = \frac{1}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

We see that the formula is **fulfilled**:

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

$\Rightarrow$

$$\left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2}\right)/32 = \frac{1}{384\pi^2}$$

Correspondence between **gravity** and **hydrodynamics** is **confirmed!**

Direct verification: spin 3/2

Rarita-Schwinger-Adler model of spin 3/2

The **Rarita-Schwinger theory** – well-known theory of spin 3/2.
But this theory has a number of **pathologies.**

- For example, it **doesn't allow to construct perturbation theory!**

Solved in **[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]** by introducing of interaction with additional spin ½ field:

Action:

additional spin ½ field

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu}\bar{\psi}_\lambda\gamma_5\gamma_\mu\partial_\nu\psi_\rho + i\bar{\lambda}\gamma^\mu\partial_\mu\lambda - im\bar{\lambda}\gamma^\mu\psi_\mu + im\bar{\psi}_\mu\gamma^\mu\lambda\right)$$

"coupling mass"

Anomaly was found in **[Prokhorov, Teryaev, Zakharov, Phys.Rev.D 106 (2022) 2, 025022]**

$$\nabla_\mu j^\mu_A = -\frac{19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

-19 times different from the anomaly for spin ½

Zubarev density operator

Global Equilibrium Conditions

$$\nabla_\mu \beta_\nu + \nabla_\nu \beta_\mu = 0 \qquad \nabla_\mu \zeta = 0$$

Form of the density operator for a medium with rotation and acceleration

$$\widehat{\rho} = \frac{1}{Z} \exp\left[-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu} + \zeta \widehat{Q}\right]$$

Thermal vorticity tensor

Lorentz Transform Generators

$$\widehat{J}^{\mu\nu} = \int_\Sigma d\Sigma_\lambda \left(x^\mu \widehat{T}^{\lambda\nu} - x^\nu \widehat{T}^{\lambda\mu}\right)$$

$$\varpi_{\mu\nu}\hat{J}^{\mu\nu} = -2\alpha^\rho \hat{K}_\rho - 2w^\rho \hat{J}_\rho$$

$\hat{K}^\mu$ – boost (related to **acceleration**)

$\hat{J}^\mu$ – angular momentum (related to **vorticity**)

KVE IN RSA THEORY: CALCULATION

- Our *goal* is to calculate the conductivities λ_1 and λ_2 in the KVE current:

$$j^{\mu}_{A,KVE} = \lambda_1(\omega_\nu\omega^\nu)\omega^\mu + \lambda_2(a_\nu a^\nu)\omega^\mu$$

- Using the perturbation theory, we obtain:

$$\lambda_1 = -\frac{1}{6}\int_0^{|\beta|}[d\tau]\langle T_\tau \hat{J}^3_{-i\tau_x}\hat{J}^3_{-i\tau_y}\hat{J}^3_{-i\tau_z}\hat{j}^3_A(0)\rangle_{T,c}$$

- The matrix element has the form of a product of **vertices** and **propagators**.

Vertices

$$\mathcal{D}^{\mu\nu}_{(ij)} = -\frac{1}{2}(-i)^{\delta_{0\mu}+\delta_{0\nu}}\varepsilon^{ij\nu\beta}\left(\gamma_5\tilde{\gamma}_\mu\tilde{\partial}^{X_2}_\beta - \frac{1}{4}\gamma_5\tilde{\gamma}_\beta[\tilde{\gamma}_\vartheta,\tilde{\gamma}_\mu]\left(\tilde{\partial}^{X_1}_\vartheta + \tilde{\partial}^{X_2}_\vartheta\right)\right) + (\mu\leftrightarrow\nu)$$

$0 \leq (i,j) < 4$

Propagators

$$\langle T_\tau\tilde{\psi}_{a\mu}(X_1)\bar{\tilde{\psi}}_{b\nu}(X_2)\rangle_T = \sum_P e^{iP_\alpha(X_1-X_2)^\alpha}\frac{i}{2P^2}\left(\tilde{\gamma}_\nu \not{P}\tilde{\gamma}_\mu + 2\left[\frac{1}{m^2}-\frac{2}{P^2}\right]P_\mu P_\nu \not{P}\right)_{ab}$$

$$\langle T_\tau\tilde{\psi}_{a\mu}(X_1)\bar{\lambda}_b(X_2)\rangle_T = \sum_P e^{iP_\alpha(X_1-X_2)^\alpha}\frac{-P_\mu \not{P}_{ab}}{mP^2}$$

Mixed terms are non-zero

$$\langle T_\tau\lambda_a(X_1)\bar{\lambda}_b(X_2)\rangle_T = 0$$

here $P_\mu = (p_n, -\mathbf{p})\,,\ p_n = (2n+1)\pi T$

Field λ is **non-propagating**

KVE vs Gravitational Anomaly

The obtained formula for cubic gradients (KVE):

$$j_\mu^{A(3)} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2\right)\omega_\mu$$

Gravitational chiral **anomaly**:

$$\nabla_\mu j_A^\mu = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}{}^{\lambda\rho}$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$

Direct **verification**:

$$\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2}\right)\Big/ 32 = -\frac{19}{384\pi^2}$$

Coincidence of hydrodynamics and gravitational anomaly!

- The relationship between the transport coefficients in a **vortical accelerated fluid** and the **gravitational** chiral **anomaly** is shown!
- Verification in a **nontrivial** case with higher spins and interaction.

Conclusion

Conclusion

- The relationship between the hydrodynamic current in the third order of gradient expansion $\lambda_1(\omega_\nu\omega^\nu)\omega_\mu$ and $\lambda_2(a_\nu a^\nu)w_\mu$, the Kinematic Vortical Effect (KVE), and the gravitational chiral anomaly has been established.

- The obtained formula has been verified directly for spins **1/2** and **3/2**.

Additional slides

Generalization to (anti)de Sitter space

- Going **beyond approximation** $R_{\mu\nu} = 0$ [Khakimov, Prokhorov, Teryaev, Zakharov,2401.09247 (2024)]

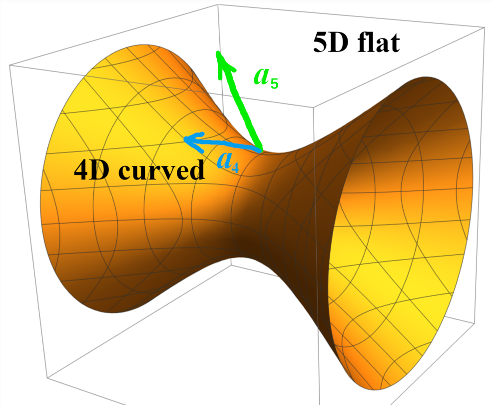
$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$ -- **anomaly-hydro** relation remains valid

$$j^A_\mu \sim a^2 \omega_\mu \iff j^A_\mu \sim R\omega_\mu$$ -- equivalence principle in higher orders

- **5-dimensional Unruh effect**: [Khakimov, Prokhorov, Teryaev, Zakharov, Phys.Rev.D 108, 12, L121701 (2023)]

The temperature measured by an accelerated observer in **(A)dS space** is determined by the 5-dimensional acceleration!

[S. Deser and O. Levin, Phys. Rev. D, 59:064004, 1999]



Hydrodynamic expansion for the stress-energy tensor:

$$\langle \hat{T}^{\mu\nu} \rangle_{s=1/2} = \left[\frac{7\pi^2}{180} T^4 + \frac{1}{72} \left(|a|^2 + \frac{R}{12} \right) T^2 - \frac{17}{2880\pi^2} \left(|a|^2 + \frac{R}{12} \right)^2 \right] \left(4u^\mu u^\nu - g^{\mu\nu} \right) + \frac{11}{960\pi^2} \left(\frac{R}{12} \right)^2 g^{\mu\nu}$$

$$\langle \hat{T}^{\mu\nu} \rangle (T = T_{UR}) = \frac{k}{4} R^2 g^{\mu\nu}$$ has a vacuum form $$T_{UR} = \frac{a_5}{2\pi} = \sqrt{T_U^2 + T_R^2} = \frac{\sqrt{|a|^2 + R/12}}{2\pi}$$

Rarita-Schwinger-Adler model of spin 3/2

The **Rarita-Schwinger theory** – well-known theory of spin 3/2.
But this theory has a number of **pathologies.**

Generalized Hamiltonian dynamics: **Dirac bracket** instead of **Poisson bracket**

$$[F(\vec{x}), G(\vec{y})]_D = [F(\vec{x}), G(\vec{y})] - \int d^3w d^3z [F(\vec{x}), \chi^\dagger(\vec{w})] M^{-1}(\vec{w}, \vec{z}) [\chi(\vec{z}), G(\vec{y})]$$

$$M(\vec{x}, \vec{y}) = [\chi(\vec{x}), \chi^\dagger(\vec{y})]$$

There is **singularity** in a Dirac bracket in weak gauge field limit for RS-theory!

Doesn't allow to construct perturbation theory!

Solved in **[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]** by introducing of interaction with additional spin ½ field:

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu \partial_\nu \psi_\rho + i\bar{\lambda}\gamma^\mu \partial_\mu \lambda - im\bar{\lambda}\gamma^\mu \psi_\mu + im\bar{\psi}_\mu \gamma^\mu \lambda \right)$$

Rarita-Schwinger-Adler model of spin 3/2

- The interaction **shifts the pole** in the Dirac bracket!

$$[\Psi_i(\vec{x}), \Psi_j^\dagger(\vec{y})]_D = -\, i \left[(\delta_{ij} - \frac{1}{2}\sigma_i\sigma_j)\delta^3(\vec{x}-\vec{y}) - \overrightarrow{D}_{\vec{x}\, i} \frac{\delta^3(\vec{x}-\vec{y})}{m^2 + g\vec{\sigma}\cdot\vec{B}(\vec{x})} \overleftarrow{D}_{\vec{y}\, j} \right]$$

Contribution of interaction with an additional field

- The **stress-energy tensor** can be obtained by varying with respect to the metric

$$T^{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}$$

$$T^{\mu\nu} = \frac{1}{2}\varepsilon^{\lambda\nu\beta\rho}\bar{\psi}_\lambda\gamma_5\gamma^\mu\partial_\beta\psi_\rho + \frac{1}{8}\partial_\eta\left(\varepsilon^{\lambda\alpha\nu\rho}\bar{\psi}_\lambda\gamma_5\gamma_\alpha[\gamma^\eta,\gamma^\mu]\psi_\rho\right) + \frac{i}{4}\left(\bar{\lambda}\gamma^\nu\partial^\mu\lambda - \partial^\mu\bar{\lambda}\gamma^\nu\lambda\right) + \frac{i}{2}m\left(\bar{\psi}^\mu\gamma^\nu\lambda - \bar{\lambda}\gamma^\nu\psi^\mu\right) + (\mu \leftrightarrow \nu)\,.$$

Traceless unlike the usual Rarita-Schwinger field $T^\mu_\mu = 0$

- The currents can be obtained from Noether's theorem. The **axial current** can be constructed for the $U(1)_A$ transformation:

$$j^\mu_A = -i\varepsilon^{\lambda\rho\nu\mu}\bar{\psi}_\lambda\gamma_\nu\psi_\rho + \bar{\lambda}\gamma_\mu\gamma_5\lambda$$

Chiral anomaly in RSA theory: gauge part

- Since the problem with the Dirac bracket is solved – **perturbation theory** can be constructed

- The **chiral (gauge) quantum anomaly** was obtained by the shift method:

[Stephen L. Adler. Phys. Rev. D, 97(4):045014, 2018]

$$\langle \partial_\mu \hat{j}_A^\mu \rangle = -\frac{5}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

see also

[S. L. Adler, P. Pais, Phys. Rev. D 99, 095037 (2019)]

Also by the method of conformal three-point functions:

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 106 (2) (2022) 025022]

- The factor "**5**" differs from what is expected according to the prediction "**3**" based on supergravity

[M. J. Duff, in First School on Supergravity (1982) arXiv:1201.0386]

However the correspondence is restored if we take into account that there are two additional degrees of freedom with spin ½: then ***5=3+2***

Gravitational chiral anomaly in RSA theory

For a conformally symmetric theory, if

$$\partial_\mu T^{\mu\nu} = 0\,, \quad \partial_\mu j^\mu_V = 0\,, \quad \partial_\mu j^\mu_A = 0\,,$$
$$T^\mu_\mu = 0\,, \quad T_{\mu\nu} = T_{\nu\mu}\,.$$

It is proven in [J. Erdmenger.Nucl. Phys. B, 562:315–329, 1999], that the three-point function $\langle T\hat{T}_{\mu\nu}(x)\hat{T}_{\sigma\rho}(y)\hat{j}^A_\omega(z)\rangle_c$ has the **universal form**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^\omega_A(z)\rangle_c = \frac{1}{(x-z)^8(y-z)^8}$$
$$\times \mathscr{I}_T^{\mu\nu,\mu'\nu'}(x-z)\mathscr{I}_T^{\sigma\rho,\sigma'\rho'}(y-z)t^{TTA}_{\mu'\nu'\sigma'\rho'}{}^{\omega}(Z)$$

where the notations are introduced:

$$\mathscr{I}^T_{\mu\nu,\sigma\rho}(x) = \mathscr{E}^T_{\mu\nu,\alpha\beta} I^\alpha_\sigma(x) I^\beta_\rho(x)\,,$$
$$\mathscr{E}^T_{\mu\nu,\alpha\beta} = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{4}\eta_{\mu\nu}\eta_{\alpha\beta}\,,$$
$$t^{TTA}_{\mu\nu\sigma\rho\omega}(Z) = \frac{\mathscr{A}}{Z^6}(\mathscr{E}^T_{\mu\nu,\eta}{}^{\varepsilon}\mathscr{E}^T_{\sigma\rho,\kappa\varepsilon}\varepsilon_\omega{}^{\eta\kappa\lambda}Z_\lambda$$
$$-6\,\mathscr{E}^T_{\mu\nu,\eta\gamma}\mathscr{E}^T_{\sigma\rho,\kappa\delta}\varepsilon_\omega{}^{\eta\kappa\lambda}Z^\gamma Z^\delta Z_\lambda Z^{-2})$$

"6" – consequence of $T^\mu_\mu = 0$

Gravitational chiral anomaly in RSA theory

[G. Y. Prokhorov, O. V. Teryaev, V. I. Zakharov, Phys. Rev. D 106 (2) (2022) 025022]

Summing 9 correlates (contributions of different), we will obtain:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle_c = -19\Big(4\pi^6(x-y)^5 \times(x-z)^3(y-z)^3\Big)^{-1} e_{\vartheta}\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^{\nu}e^{\rho}\varepsilon^{\sigma\vartheta\mu\omega} +e^{\mu}e^{\rho}\varepsilon^{\sigma\vartheta\nu\omega}+e^{\sigma}e^{\nu}\varepsilon^{\vartheta\mu\rho\omega}+e^{\sigma}e^{\mu}\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

$\Longleftrightarrow$

Matches the form we want!

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}^{\omega}_{A}(z)\rangle = \mathscr{A}\Big(4(x-y)^5 \times(x-z)^3(y-z)^3\Big)^{-1} e_{\vartheta}\Big(\eta^{\nu\rho}\varepsilon^{\sigma\vartheta\mu\omega}+\eta^{\mu\rho}\varepsilon^{\sigma\vartheta\nu\omega} +\eta^{\nu\sigma}\varepsilon^{\vartheta\mu\rho\omega}+\eta^{\mu\sigma}\varepsilon^{\vartheta\nu\rho\omega}-6e^2(e^{\nu}e^{\rho}\varepsilon^{\sigma\vartheta\mu\omega} +e^{\mu}e^{\rho}\varepsilon^{\sigma\vartheta\nu\omega}+e^{\sigma}e^{\nu}\varepsilon^{\vartheta\mu\rho\omega}+e^{\sigma}e^{\mu}\varepsilon^{\vartheta\nu\rho\omega})\Big)$$

(points on the same axis)

We can determine the factor in the anomaly:

$$\mathscr{A}_{RSA} = -19\mathscr{A}_{s=1/2} = -\frac{19}{\pi^6}$$

$$\langle\nabla_{\mu}\hat{j}^{\mu}_{A}\rangle_{RSA} = \frac{-19}{384\pi^2\sqrt{-g}}\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\kappa\lambda}R_{\rho\sigma}{}^{\kappa\lambda}$$

–19 times different from the anomaly for spin ½

Gravitational chiral anomaly in RSA theory

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_\mu \hat{j}^\mu_A \rangle_{RS} = \frac{-21}{384\pi^2\sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

"ghostless" contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

Contribution of spin 1/2

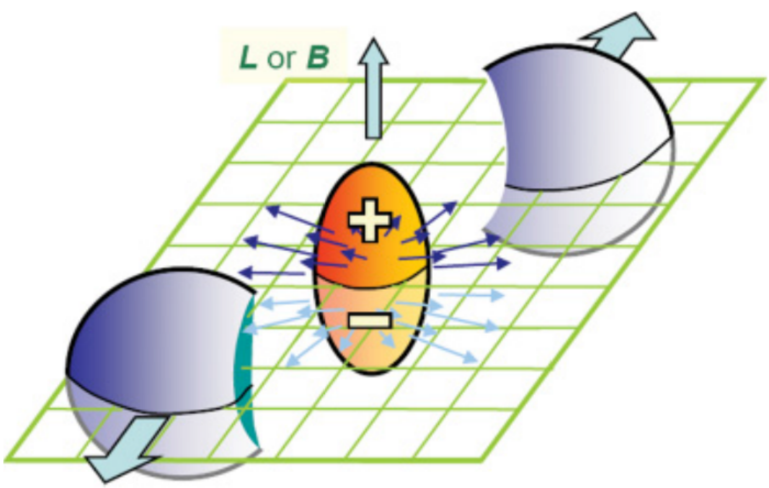
$$-19 = -21 + 2$$

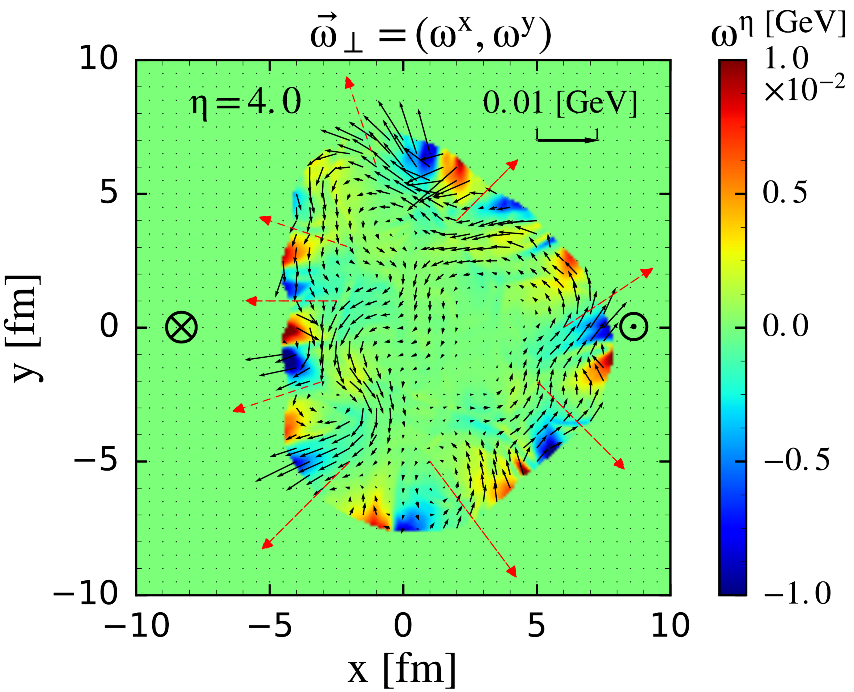
EXPERIMENT: FEW WORDS

In non-central collisions of heavy ions, huge magnetic fields and huge angular momentum arise. Differential rotation - different at different points: **vorticity** and **vortices**.

– Rotation 25 orders of magnitude faster, than the rotation of the earth:

– The vorticity has order 10^{22} sec^{-1}

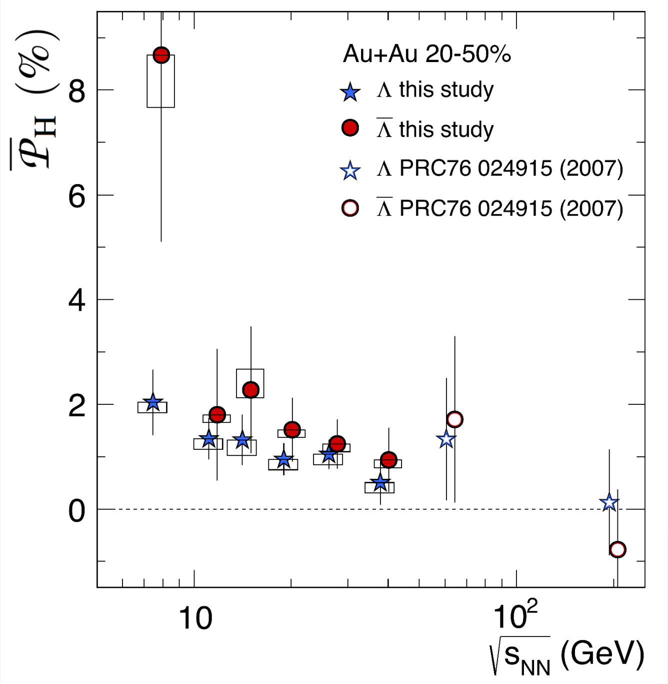




[Phys. Rev. Lett. 117, 192301 (2016)]

Experiment: few words

Vorticity transforms into polarization



[Nature 548 (2017) 62-65 arXiv:1701.06657 [nucl-ex]]

- Generation of **hyperon polarization**.
- Both **vorticity** and **acceleration** are essential for polarization.
- Also described based on **Chiral Vortical Effect** (**CVE**) **[Rogachevsky, Sorin, Teryaev, Phys.Rev.C82(2010) 054910]**, **[Baznat, Gudima, Sorin, Teryaev, Phys.Rev.C93, no.3,031902 (2016)]**

CVE: $$\langle j^5_\mu \rangle = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2}\right)\omega_\mu$$

– **Qualitative and quantitative correspondence!**

– Polarization from quantum anomaly ~ *spin crisis* and *gluon anomaly*: **[Efremov, Soffer, Teryaev, Nucl.Phys.B 346 (1990) 97-114]**

proton spin → hyperon polarization,
gluon field → chemical potential*4-velocity

Also described on the basis of a thermodynamic approach (Wigner function):

[I. Karpenko, F. Becattini, Nucl.Phys.A 982 (2019) 519-522]

Experiment: few words

- Is it possible to observe KVE in experiment?
- Is it possible to observe a **gravitational chiral anomaly** in the hydrodynamics of the matter, produced in heavy ion collisions?
- To observe cubic terms, it is necessary that the gradients (acceleration, vorticity, magnetic field...) give a contribution at least of the order of temperature

$$\omega, a \sim (0.1 - 2)T$$

[A. Zinchenko, A. Sorin, O. Teryaev, M. Baznat, J.Phys.Conf.Ser. 1435 (2020)]

[F. Becattini, M. Buzzegoli, G. Inghirami, I. Karpenko, A. Palermo, Phys.Rev.Lett. 127 (2021) 27, 272302]

However, the cubic terms are suppressed by the numerical factor

KVE: $$j^{\mu}_{A,S=1/2} = \left(-\frac{1}{24\pi^2}\omega^2 - \frac{1}{8\pi^2}a^2 \right)\omega^{\mu}$$

The **good** news: for spin 3/2 it is enhanced by cubic growth with spin:

The **bad** news: should be suppressed by mass $e^{-m/T}$ (omega baryon is heavy).

Idea: consider massless **quasiparticles** with spin 3/2 in semymetals?

[I. Boettcher, Phys. Rev. Lett. 124, 127602 (2020)]

Modern development and the problem

A whole set of similar effects has been found at the intersection of quantum field theory and hydrodynamics **[M. N. Chernodub et al. 2110.05471]**.

Experimental search is in progress:

- CME not yet found in ion collisions at RHIC **[D.E. Kharzeev et al. 2205.00120]**.
- Condensed matter copies of the effects are found in semimetals

  [Qiang Li et al. Nature Phys. 12 (2016)].