International School of Subnuclear Physics

News from the Four Interactions

Mapping the Universe's Expansion with Gravitational Waves and Neutral Hydrogen

June 20th, 2024

Erice, Sicilia

GRAN SASSO SCIENCE INSTITUTE
SCHOOL OF ADVANCED STUDIES
Scuola Universitaria Superiore

Ulyana Dupletsa

What's so special about H ?

FLRW metric

$$ds^2 = dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin\theta^2 d\phi^2\right)\right]$$

$$H(t) = \frac{\dot{a}}{a}$$

Hubble Parameter

Hubble Constant

$$H_0 = H(\text{today})$$

$$h = H_0/100 \text{ kms}^{-1}\text{Mpc}^{-1}$$

What's so special about

$$H$$

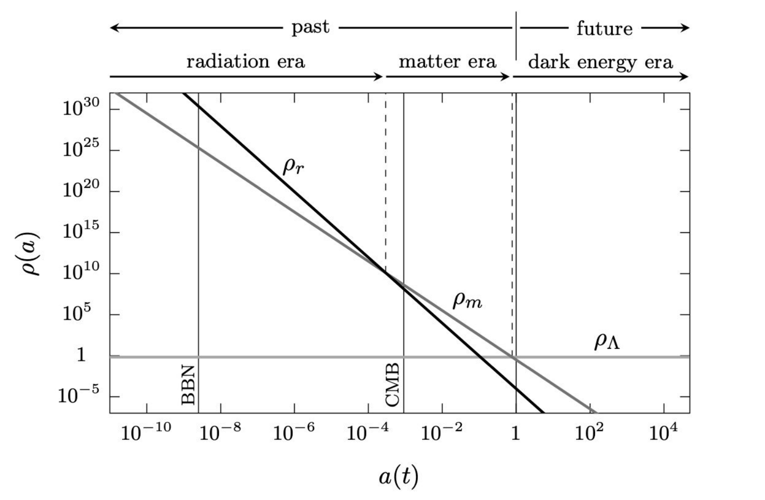
?

- **Age** of the Universe H_0
- **Size** of the Universe H_0
- The current **expansion rate** of the Universe H_0
- The **composition of the Universe** $H(z)$

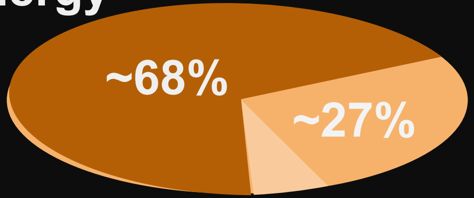
$$H(z) = H_0\sqrt{\Omega_\mathrm{m}(1+z)^3 + \Omega_\Lambda + \cancel{\Omega_\mathrm{r}(1+z)^4} + \cancel{\Omega_\mathrm{k}(1+z)^2}}$$

dark matter | **dark energy** | radiation | curvature

$$\Omega_i = \frac{\rho_i}{\rho_\mathrm{crit}}$$



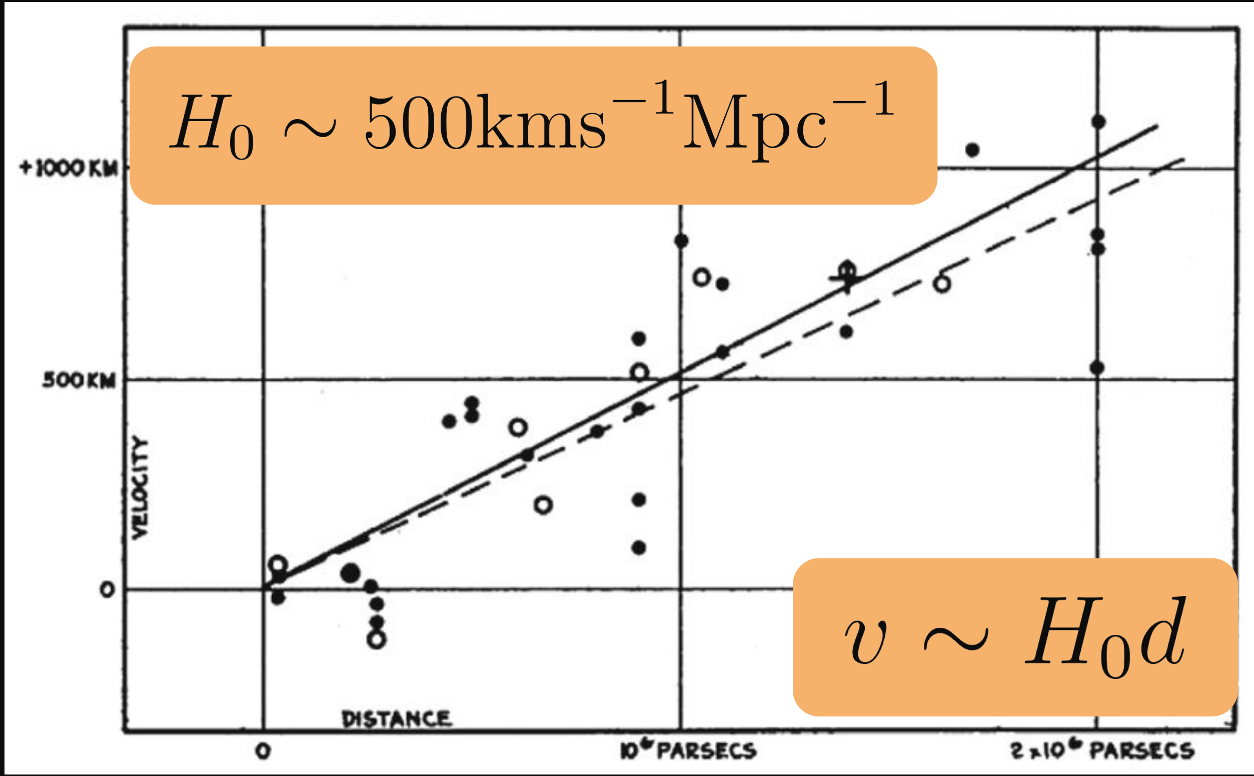
[Baumann *Modern Cosmology*]



Measuring the expansion: a **distance-redshift relation**

$$d_{\mathrm{L}} = c(1+z)\int_0^z \frac{dz'}{H(z')}$$

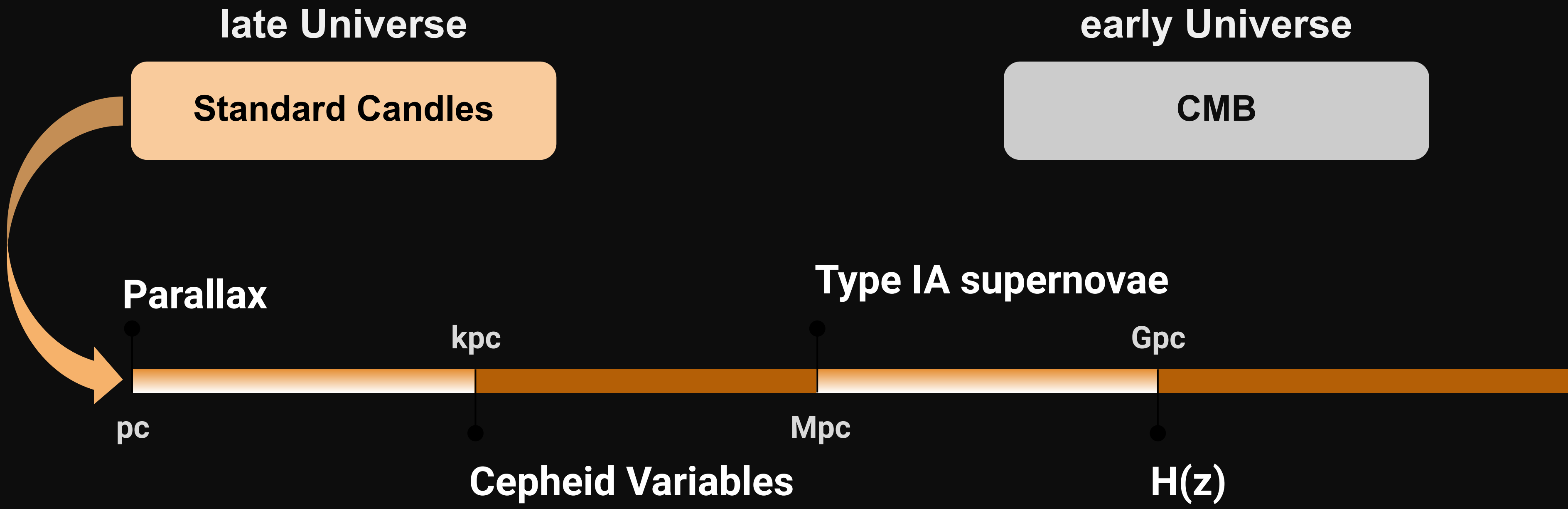
[Hubble 1929]

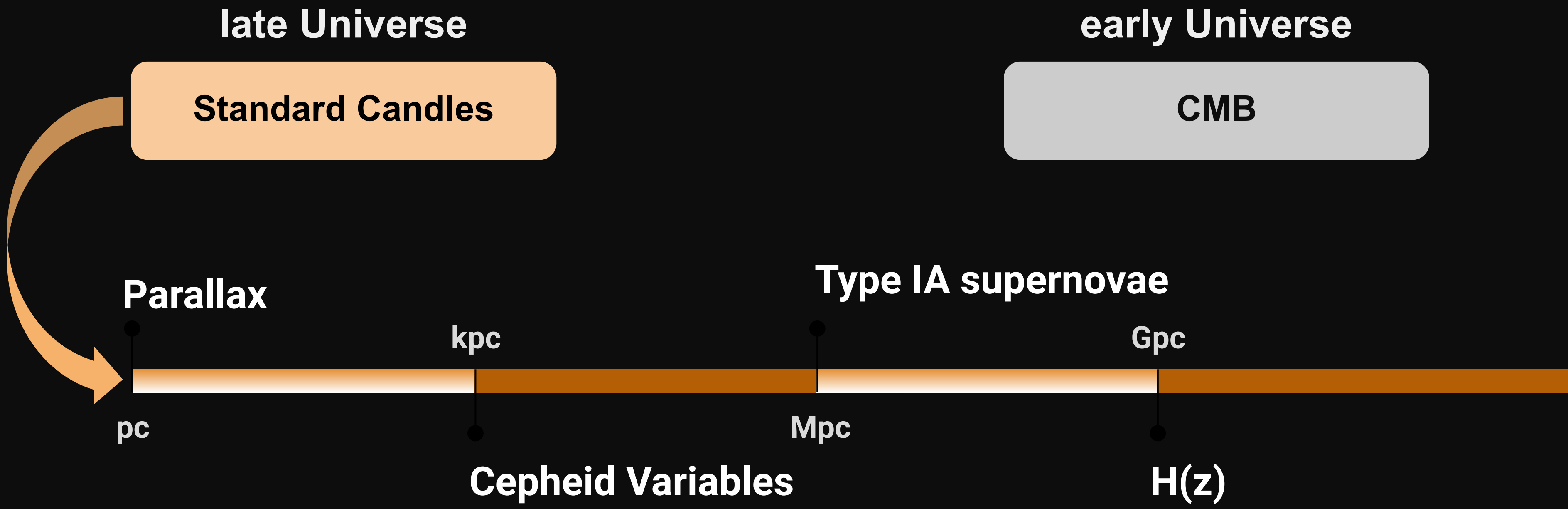


$$H_0 \sim 500 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$$

$$v \sim H_0 d$$

Hubble Law


late Universe
Standard Candles
early Universe
CMB
Parallax
Type IA supernovae
kpc
Gpc
pc
Mpc
Cepheid Variables
H(z)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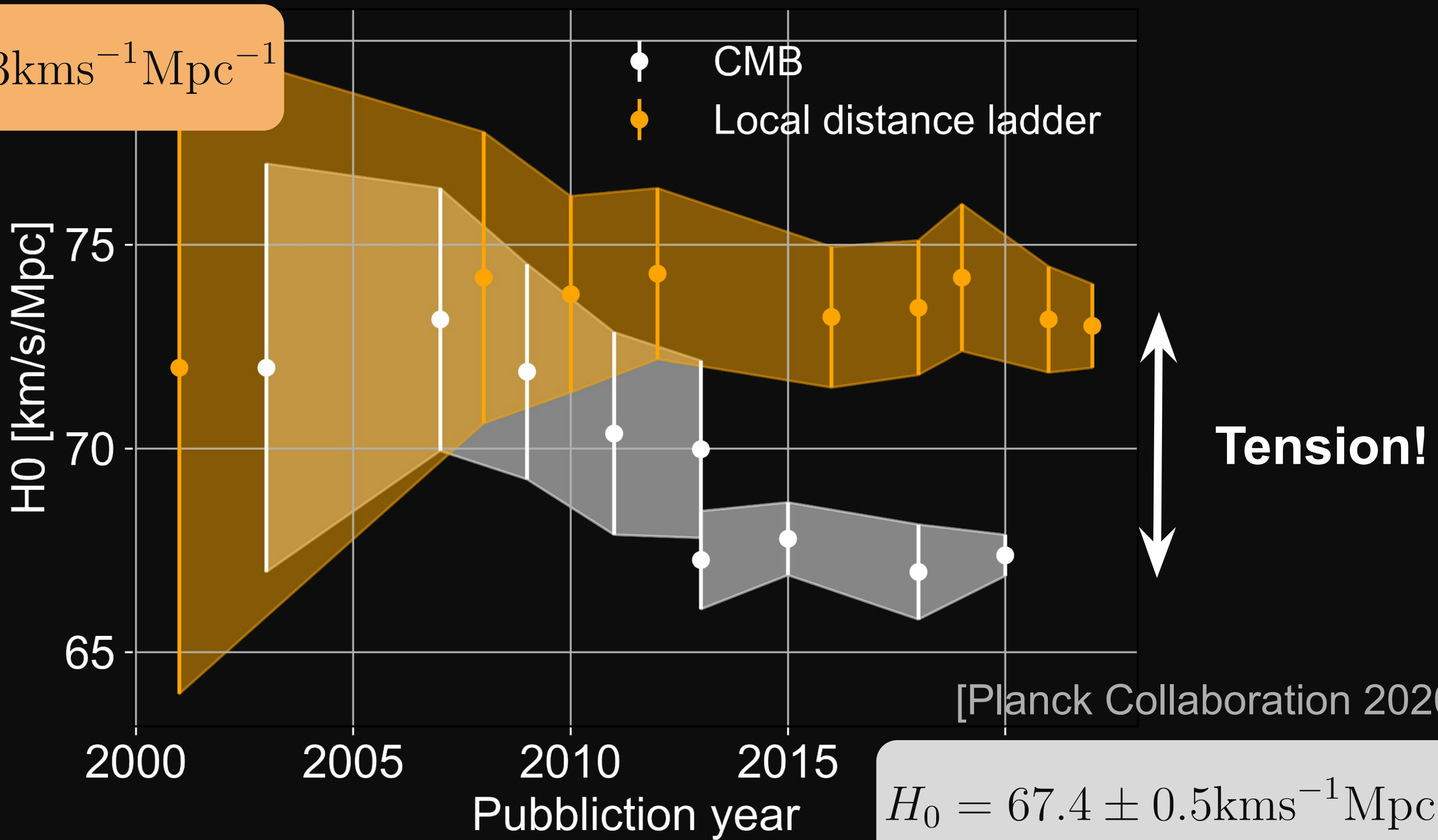
$$d_L = (1 + z^2) d_A$$

[Riess et al. 2021]

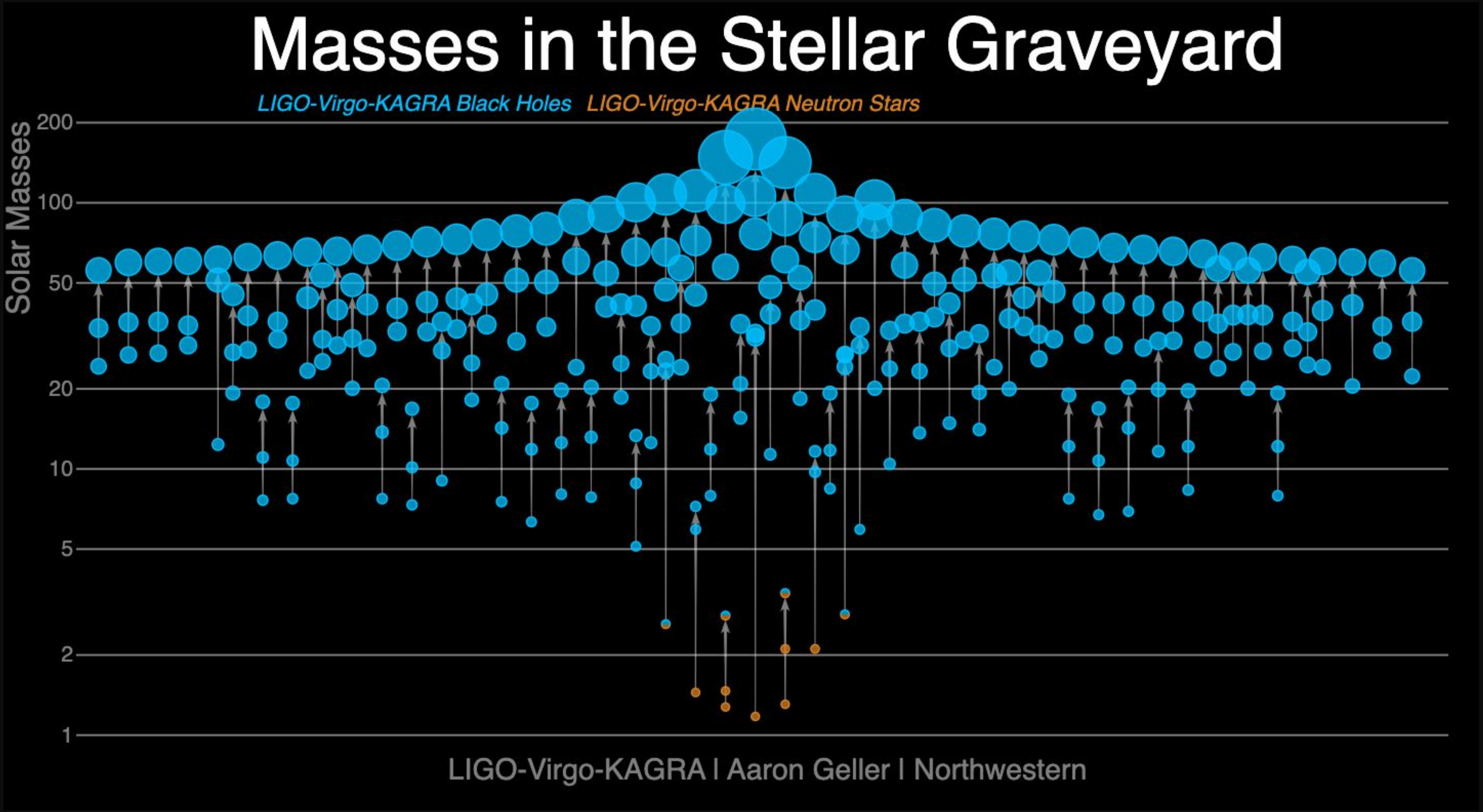
$$H_0 = 73.2 \pm 1.3 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$$



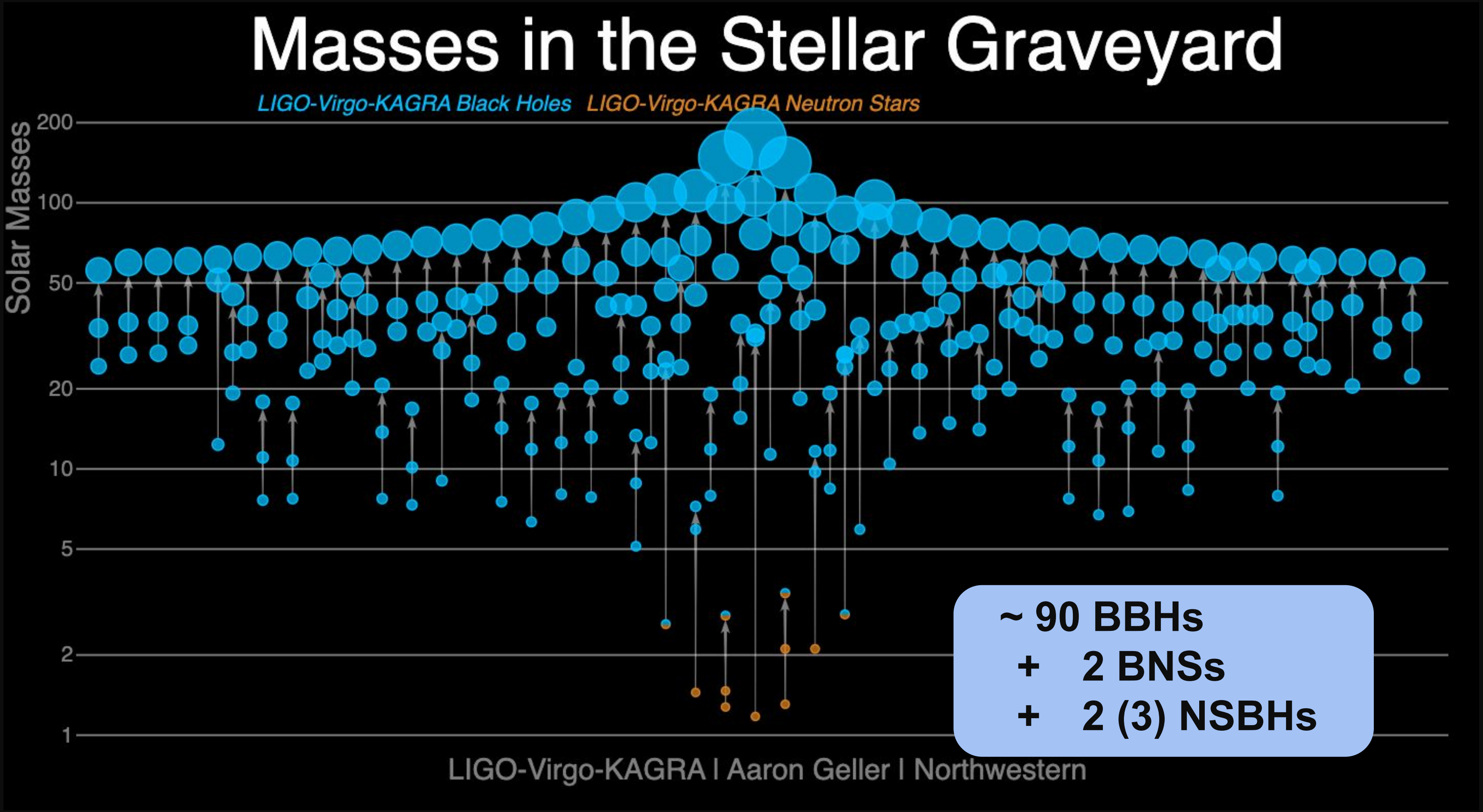
[Planck Collaboration 2020]

$$H_0 = 67.4 \pm 0.5 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$$

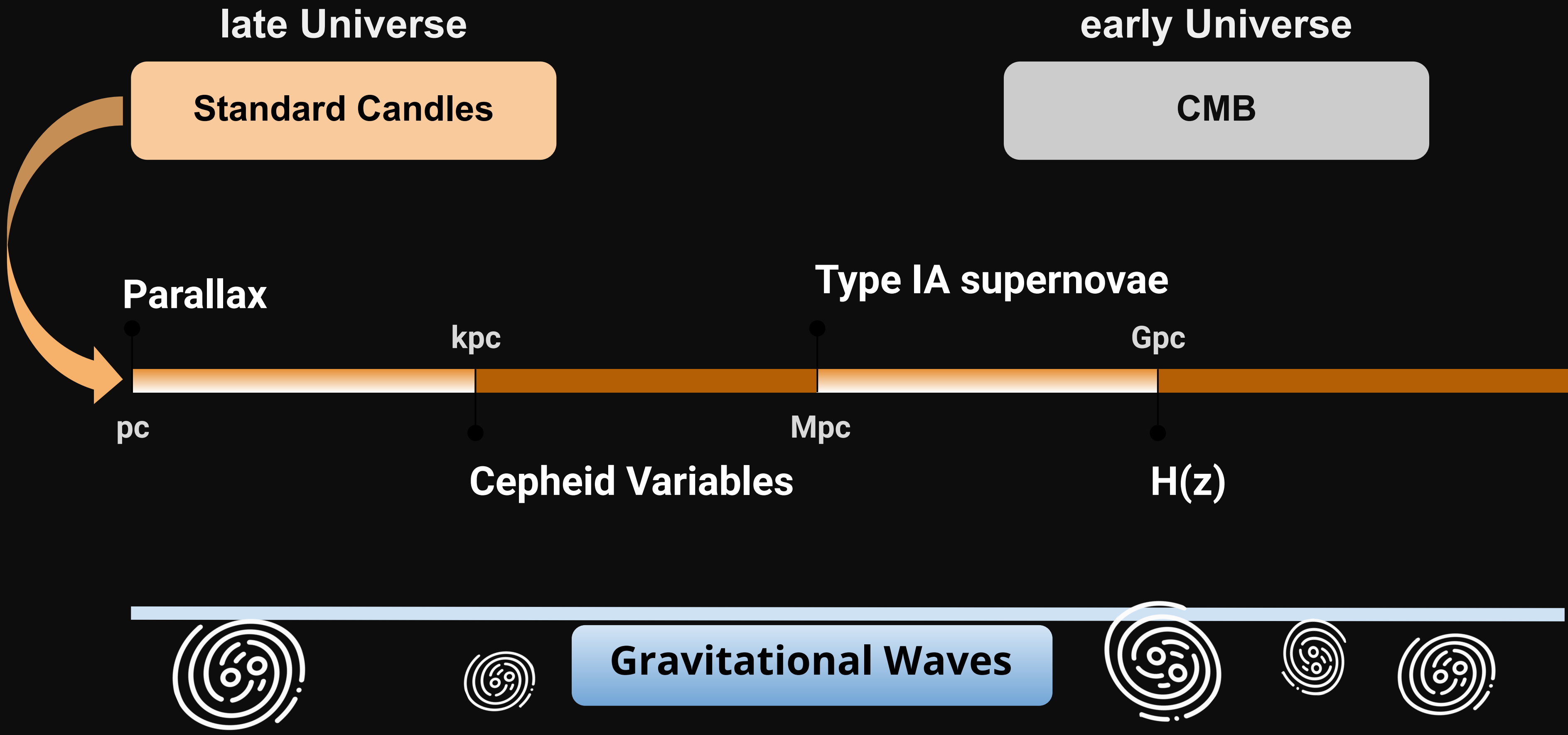
[Data from review paper: ArXiv:2105208]



Masses in the Stellar Graveyard
LIGO-Virgo-KAGRA Black Holes
LIGO-Virgo-KAGRA Neutron Stars
Solar Masses
200
100
50
20
10
5
2
1
LIGO-Virgo-KAGRA | Aaron Geller | Northwestern



~ 90 BBHs
+ 2 BNSs
+ 2 (3) NSBHs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Cepheid Variables
H(z)
Gravitational Waves

[Schutz 1986]

Standard sirens

- Gravitational waves are self-calibrated distance indicators: **cosmic rulers**

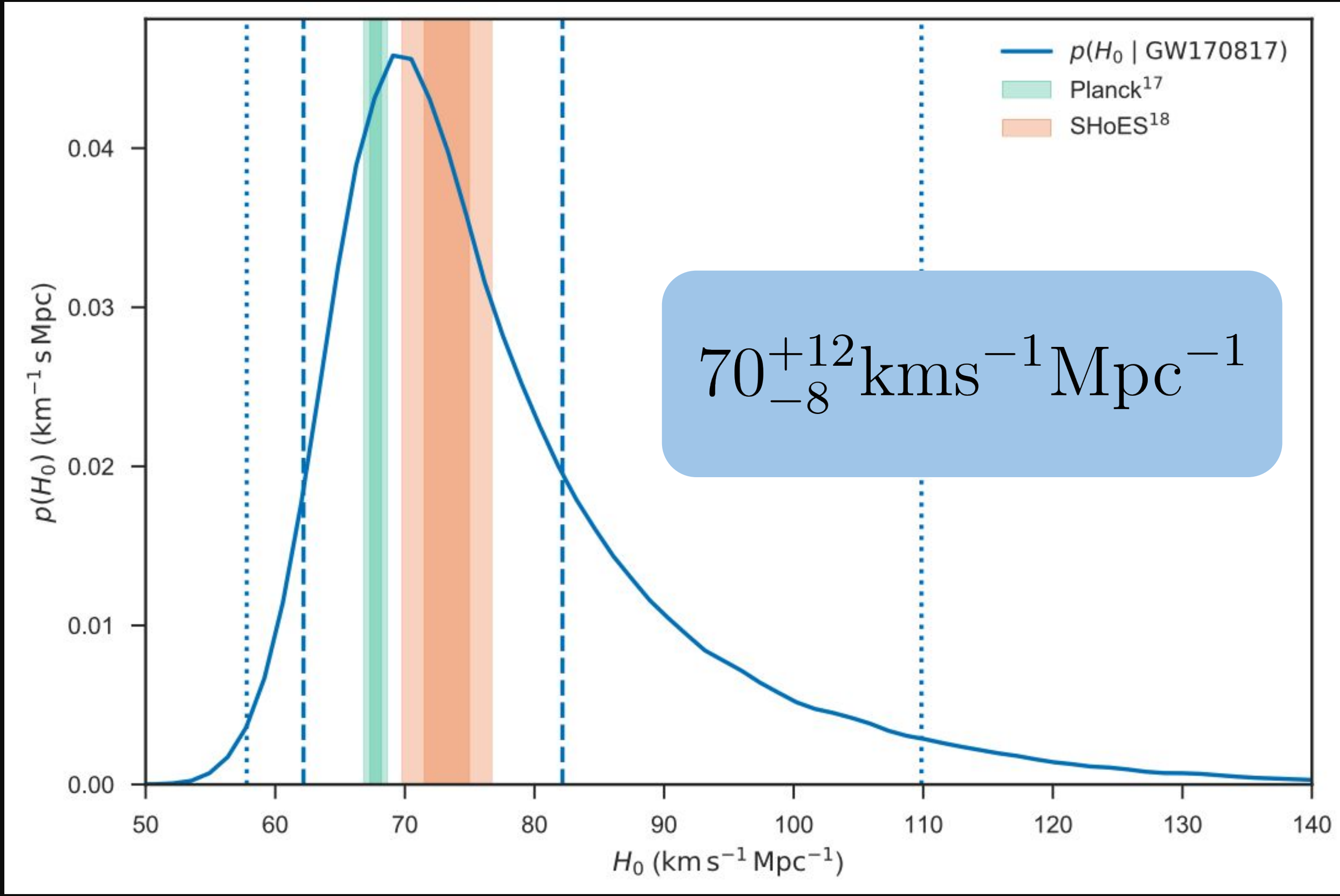
$$h_+ \propto \frac{c}{d_\mathrm{L}} \left(\frac{G\mathcal{M}_z}{c^3} \right)^{5/6} \frac{1}{f^{7/6}} \left(\frac{1 + \cos^2 \iota}{2} \right) e^{i\Psi_+}$$

$$h_\times \propto \frac{c}{d_\mathrm{L}} \left(\frac{G\mathcal{M}_z}{c^3} \right)^{5/6} \frac{1}{f^{7/6}} \cos \iota e^{i\Psi_\times}$$

- **No direct redshift measurement** from the gravitational signal

- Methods based on complementary observations:
  - Direct EM counterpart with GW170817 (**bright sirens**)
  - Statistical association with galaxy catalogs (**dark sirens**)

- Methods based on astrophysical models:
  - Source-frame mass modeling



[LVK Collaboration 2017]

- Methods based on complementary observations:
  - Direct EM counterpart with GW170817 (**bright sirens**)
  - Statistical association with galaxy catalogs (**dark sirens**)

- Methods based on astrophysical models:
  - Source-frame mass modeling

- Methods based on complementary observations:
  - Direct EM counterpart with GW170817 (**bright sirens**)
  - Statistical association with galaxy catalogs (**dark sirens**)

- Methods based on astrophysical models:
  - Source-frame mass modeling (**spectral sirens**)

Standard sirens

- Gravitational waves are self-calibrated distance indicators: **cosmic rulers**

$$h_+ \propto \frac{c}{d_\mathrm{L}} \left(\frac{G\mathcal{M}_z}{c^3} \right)^{5/6} \frac{1}{f^{7/6}} \left(\frac{1+\cos^2 \iota}{2} \right) e^{i\Psi_+}$$

$$h_\times \propto \frac{c}{d_\mathrm{L}} \left(\frac{G\mathcal{M}_z}{c^3} \right)^{5/6} \frac{1}{f^{7/6}} \cos \iota e^{i\Psi_\times}$$

- **No direct redshift measurement** from the gravitational signal

$$H_0 = 67.4 \pm 0.5 \mathrm{km s^{-1} Mpc^{-1}}$$

	Without GW170817	With GW170817
Bright Siren	$70^{+12}_{-8} \mathrm{km s^{-1} Mpc^{-1}}$	
Spectral Sirens	$50^{+37}_{-30} \mathrm{km s^{-1} Mpc^{-1}}$	$68^{+12}_{-8} \mathrm{km s^{-1} Mpc^{-1}}$
Dark Sirens (galaxy catalogs)	$67^{+13}_{-8} \mathrm{km s^{-1} Mpc^{-1}}$	$68^{+8}_{-6} \mathrm{km s^{-1} Mpc^{-1}}$

$$H_0 = 73.2 \pm 1.3 \mathrm{km s^{-1} Mpc^{-1}}$$

[LVK Collaboration 2023]

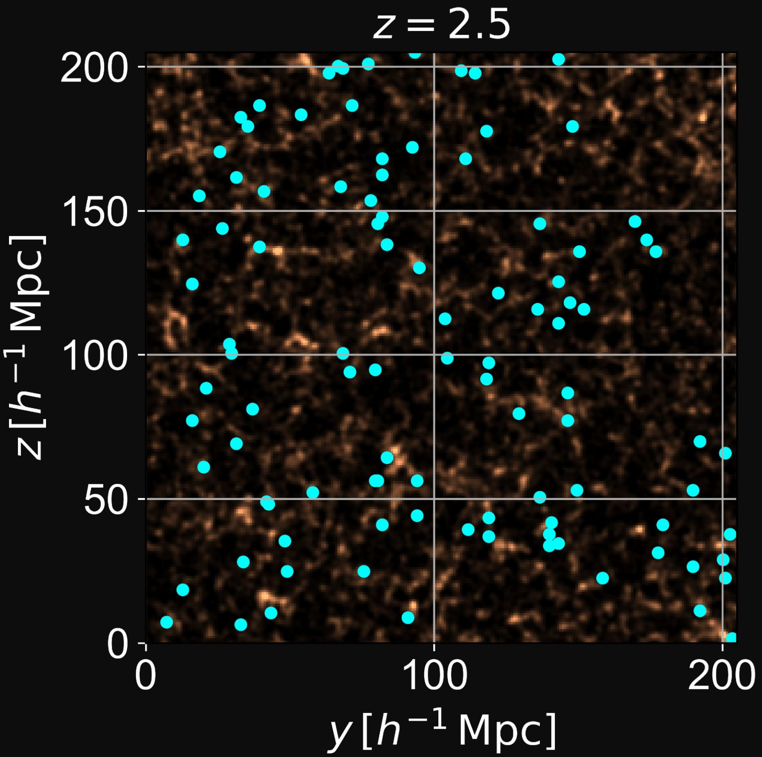
Multi-Tracing Approach

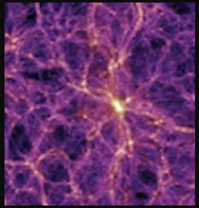
[Millennium Simulation]

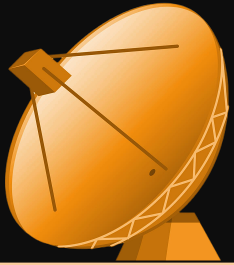
[GitHub]

Multi-Tracing Approach

BBHs and HI field follow the same LSS







Resolved GW events from stellar-mass BBHs

Next-generation GW observatories: ET (+ CE)

+

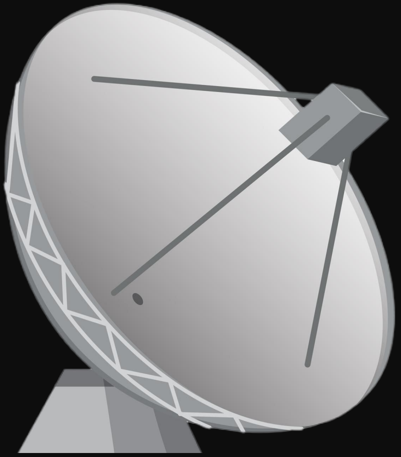
Intensity mapping of 21cm line from neutral hydrogen

Future large scale structure surveys as SKAO

→

$$H(z) = \ldots$$

[Dupletsa, Harms et al, 2022, 2024]



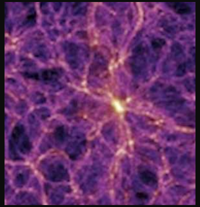
Thank You!

ulyana.dupletsa@gssi.it

Backup

Hierarchical Bayesian Inference

[Dupletsa et al, 2022, 2024]



Poisson statistics

the events are independent!

Likelihood of single GW event

$$\mathcal{L}(\{\vec{x}_i\}|\vec{\lambda}) = \frac{e^{-N_{\mathrm{exp}}(\vec{\lambda})}\left(N_{\mathrm{exp}}(\vec{\lambda})\right)^{N_{\mathrm{obs}}}}{N_{\mathrm{obs}}!}\prod_{i}^{N_{\mathrm{obs}}}\frac{\int d\vec{\theta}\,\mathcal{L}_{\mathrm{GW}}(\vec{x}_i|\vec{\theta})\,p_{\mathrm{CBC}}(z)}{\int d\vec{\theta}\,p_{\mathrm{det}}(\theta)\,p_{\mathrm{CBC}}(z)}$$

probability of detection

redshift probability

selection effects

[Mandel et al. 2019, Gair et al. 2023]

- For PE we want the evaluation of the full posterior distribution:

model parameters

$$p(\vec{\theta}|d) = \frac{p(d|\vec{\theta})p(\vec{\theta})}{p(d)}$$

prior

likelihood

$$p(d|\vec{\theta}) = \exp\left(\frac{1}{2}\langle d - h(\theta)|d - h(\theta)\rangle\right)$$

Fisher Matrix Approximation

$$p(d|\vec{\theta}) \approx \exp\left(-\frac{1}{2}\langle n - \left(\theta^i - \theta_0^i\right)\partial_{\theta^i} h(\vec{\theta}_0)|n - \left(\theta^i - \theta_0^i\right)\partial_{\theta^i} h(\vec{\theta}_0)\rangle\right)$$

$$\approx \exp\left(-\frac{1}{2}\langle n|n\rangle\right) + \exp\left(-\frac{1}{2}\Delta\theta^i \Gamma_{ij} \Delta\theta^j\right)$$

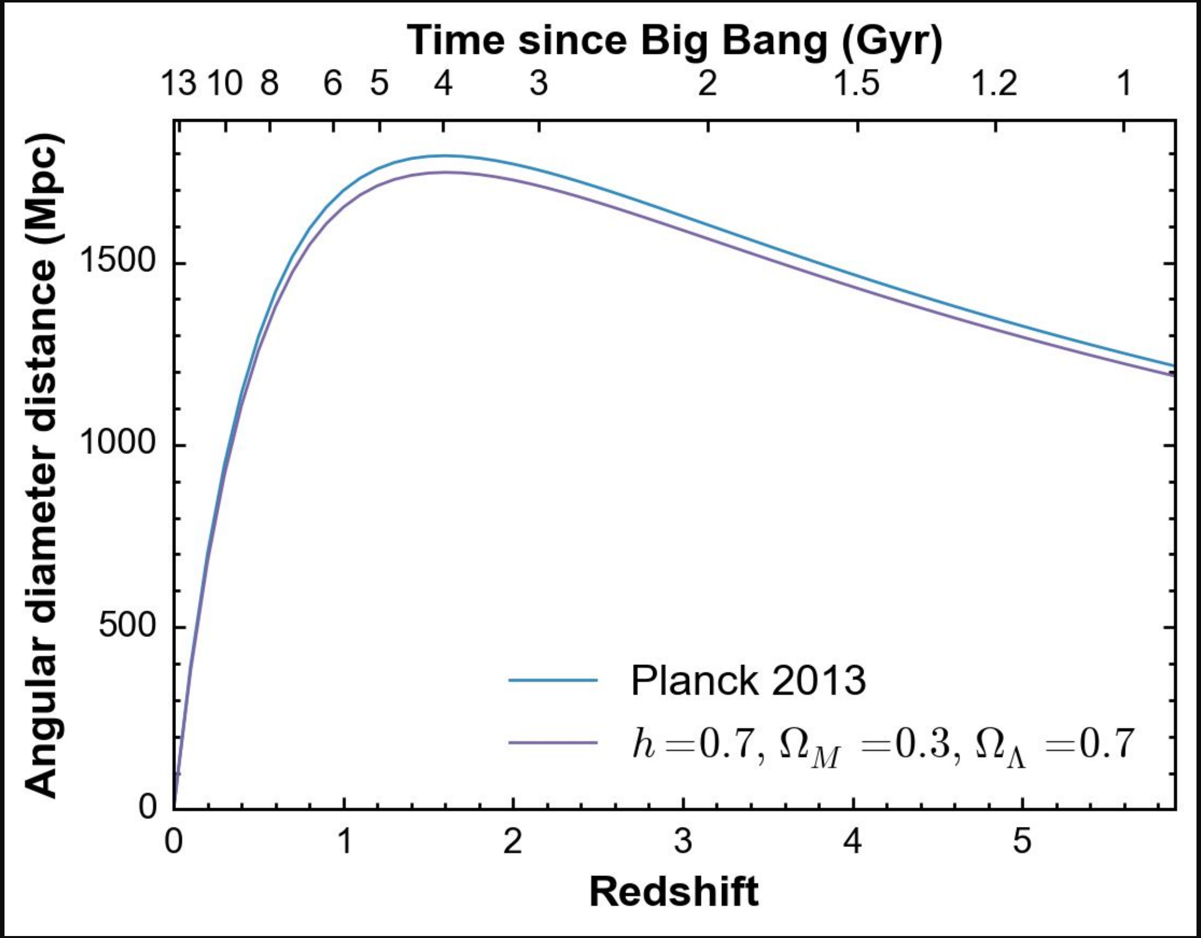
Fisher Matrix

$$\Gamma_{ij} = \langle \partial_i h(\vec{\theta})|\partial_j h(\vec{\theta})\rangle\Big|_{\theta_0}$$

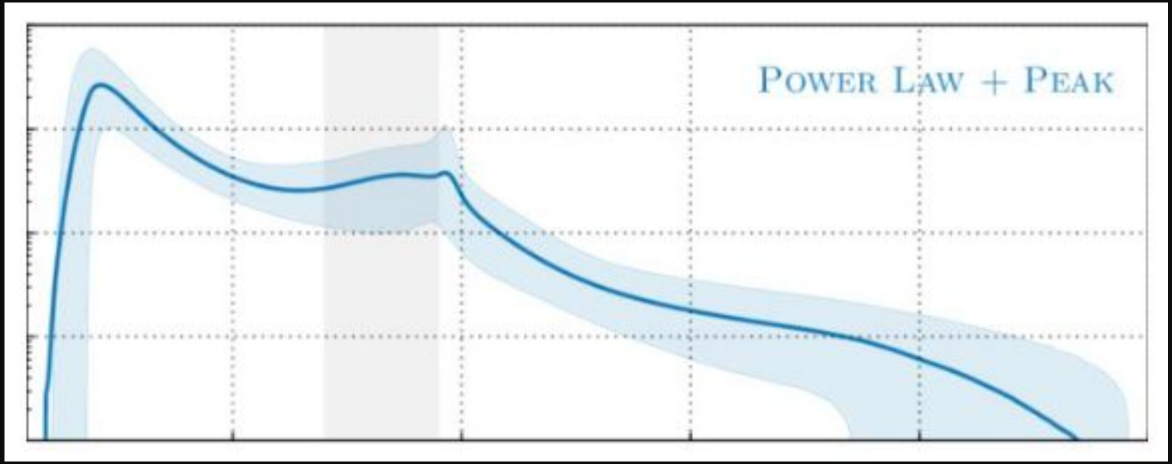
forecasts! $$\mathrm{Cov}_{ij} = \Gamma_{ij}^{-1}$$

posterior $$p(\vec{\theta}|d) = \pi(\vec{\theta})\mathcal{L}(d|\vec{\theta})$$

Fisher Matrix analysis $$e^{-\frac{1}{2}\Delta\vec{\theta}\Gamma\Delta\vec{\theta}}$$


Time since Big Bang (Gyr)
13 10 8 6 5 4 3 2 1.5 1.2 1
Angular diameter distance (Mpc)
0
500
1000
1500
0 1 2 3 4 5
Redshift
Planck 2013
$h = 0.7$, $\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$


KAGRA
Virgo
LIGO
Hanford +
Livingston



[LVK Collaboration, 2020]