Early universe holographic first order phase transition within Composite Higgs boson model

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AdS/CFT correspondence

Perturbation theory

$$\langle\phi\ldots\phi\rangle = \langle\phi_0\ldots\phi_0\rangle + \lambda\langle\phi_1\ldots\phi_1\rangle + \lambda^2\langle\phi_2\ldots\phi_2\rangle + \ldots + \text{non-analytical?} + \text{solitons?}$$

$$\text{QFT} \xrightarrow{\text{renormalization}} \text{CFT}$$

AdS/CFT correspondence

$$\text{CFT} \xleftarrow{\text{AdS/CFT}} \text{dual theory}$$

$$\mathcal{Z}_{\text{CFT}}[J] \underset{\text{correspondence}}{\overset{\text{AdS/CFT}}{=\!=}} \mathcal{Z}_{\text{AdS}} \underset{\text{approximation}}{\overset{\text{quasiclassical}}{\approx}} e^{-S_{\text{AdS}}}\Big|_{\partial\text{AdS}}$$

strong coupled $\lambda_{\text{CFT}} \gg 1$ with $\lambda_{\text{CFT}} \sim \frac{1}{\lambda_{\text{AdS}}}$
dual theory $\lambda_{\text{AdS}} \ll 1$ weak coupled

Correlators within AdS/CFT

Solutions of EoM: $\frac{\delta S_{\text{AdS}}[\psi]}{\delta\psi} = 0$

Near the conformal boundary: $\psi(x,z) \xrightarrow[z\to 0]{\partial\text{AdS}} z^{d-\Delta}\,\psi_0(x) + z^{\Delta}\,\psi_1(x)$

CFT sources: $J \sim \psi_0(x)$ with weight $d-\Delta$

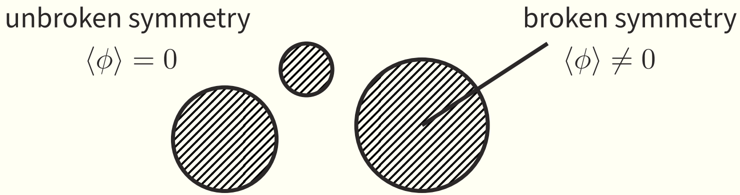
$$\text{Solutions: } \Rightarrow \mathcal{Z}_{\text{AdS}}\big|_{\partial\text{AdS}}[\psi_0] = \mathcal{Z}_{\text{CFT}}[J] = \int \mathcal{D}[\text{*fields of CFT*}]\, e^{-S_{\text{CFT}} - \mathcal{O}\cdot J}$$

$$G_n = \langle \mathcal{O}\ldots\mathcal{O}\rangle = \left(\frac{\delta}{\delta J}\right)^n \log \mathcal{Z}[J]\Big|_{J=0} = -\left(\frac{\delta}{\delta\psi_0}\right)^n S_{\text{AdS}}[\psi] = \underbrace{-\left(\frac{\delta^n S^{\text{bulk}}_{\text{AdS}}}{\delta\psi_0^n}\right)}_{=0 \text{ due to EoM}} - \left(\frac{\delta^n S^{\text{border}}_{\partial\text{AdS}}}{\delta\psi_0^n}\right)$$

Electroweak baryogenesis (Motivation)

Baryon asymmetry problem – matter more than anti-matter

Sakharov's conditions $\Leftrightarrow$ first order phase transition (i.e. CPT-violation)



$\mathsf{SU}(2)_{\mathsf{L}} \times \mathsf{U}(1)_{\mathsf{Y}} \to \mathsf{U}(1)_{\mathsf{em}}$ is a crossover (not a PT); BSM physics?

Composite Higgs model

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{CH}} + \mathcal{L}_{\text{Int.}}, \qquad \mathcal{L}_{\text{CH}}$$ – strongly coupled with $\mathcal{G}$ inner symmetry

$$\left(\mathcal{G}\begin{array}{c}\text{invariant}\\ \text{vacuum}\end{array}\right) \xrightarrow[\text{breaking}]{\text{spontaneous}} \left(\mathcal{H}\begin{array}{c}\text{invariane}\\ \text{vacuum}\end{array}\right) \Rightarrow \begin{array}{c}\text{Goldstone bosons} \ni \text{Higgs boson}\\ \text{phase transition}\end{array}$$

$$\Sigma_{IJ} = \langle \bar{\Psi}_I \Psi_J \rangle = \xi^\top \left[\begin{pmatrix} 0_{4\times4} & 0 \\ 0 & \varsigma \end{pmatrix} + \eta_i \tilde{T}_i \right] \xi \xrightarrow[\text{low energy}]{\text{SO(5)}\to\text{SO(4)}} \begin{pmatrix} 0_{4\times4} & 0 \\ 0 & \varsigma \end{pmatrix} \Rightarrow \begin{array}{c}\text{symmetry}\\ \text{breaking}\end{array}$$

Σ_{IJ} is a condensate of the $SO(5)$-inn.sym. fundamental fields Ψ;
ξ is NB-bosons, η is “radial” fluctuations, ς is background field

Holographic Model

Dual theory: $S_{\text{AdS}} = S_{\text{grav.}+\phi} + S_{\mathrm{X}} + S_{\text{gauge}} + S_{\text{“SM”}} + S_{\text{int. “SM”}+\mathrm{X}} \overset{\text{AdS/CFT}}{\sim} S_{\text{SM}+\text{CH}}$

$S_{\text{grav.}+\phi}$, S_{X} and S_{gauge} can have a PT | $S_{\text{“SM”}}$ and $S_{\text{int. “SM”}+\mathrm{X}}$ shouldn't have a PT

Matter sector: $$S_{\mathrm{X}} = \frac{1}{k_s}\int d^5x\sqrt{|g|}e^{\phi}\left[\frac{1}{2}g^{ab}\,\mathrm{Tr}\left(\nabla_a X^T \nabla_b X\right) - V_X(X)\right]$$

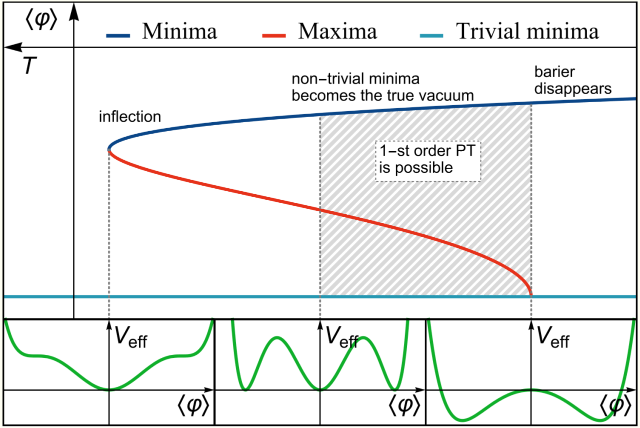
$$V_X(X) = \mathrm{Tr}\left(-\frac{3}{2L^2}X^T X - \alpha(X^T X)^2 + \beta(X^T X)^3 + \mathcal{O}(X^8)\right)$$

$$X_{IJ} \propto \frac{\sqrt{N}}{2\pi}J_{IJ}z + \frac{2\pi}{\sqrt{N}}\Sigma_{IJ}z^3 + \ldots,$$ J_{IJ} are sources for CH condensate Σ_{IJ}

Our contribution:
(and the following for the matter sector)

$$\underbrace{V_{\text{eff. CH}}[\Sigma_{IJ}]\Big|_{\text{extrema}}}_{\text{4-dimensional}} \overset{\text{AdS/CFT}}{\approx} \underbrace{S_{\mathrm{X}}[X_{IJ}; g_{ab}^{\text{fixed}}, \phi^{\text{fixed}}]\Big|_{J_{IJ}=0}^{\partial\text{AdS}}}_{\text{dual 5-dimensional}}$$

Phase transition



$$\left.\frac{\partial V_{\text{eff}}^{\text{CH}}}{\partial \langle\varphi\rangle}\right|_{\langle\varphi\rangle_0} = 0 \quad \Rightarrow \quad \left(\langle\varphi\rangle_0, V_{\text{eff}}\big|_{\langle\varphi\rangle_0}\right)$$

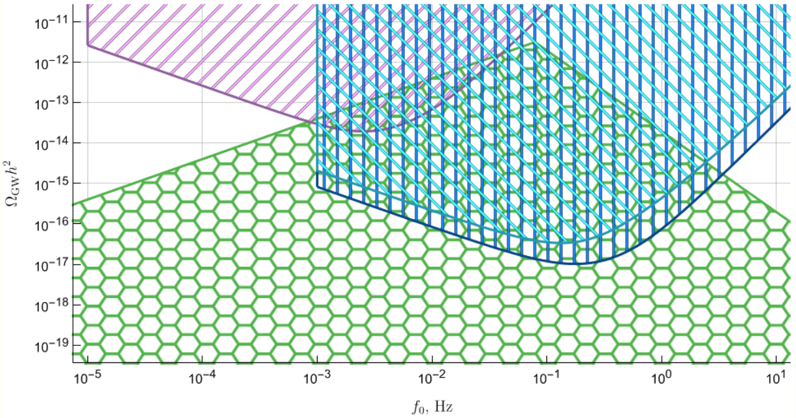
Effective potential extremal values and the positions allow one to judge about PT:

- trivial minimum (vacuum) only $\Rightarrow$ there is no PT;
- **non-trivial true vacuum with the potential barrier** $\Rightarrow$ **1-st PT**;
- non-trivial true vacuum without a potential barrier $\Rightarrow$ there is no PT.

The extrema of the effective quantum potential V_{eff}: T is the plasma temperature, $\langle\varphi\rangle$ is the vacuum expectation.

Unscaled schematic illustration! Data in real scale are Phys. Rev. D 108, 115011.

Observations



Picks of the GW spectrum estimated within Holographic Composed Higgs model. There is only the scalar part produced during initial collisions of the bubble walls (i.e. sound and turbulence contributions are not currently included in the rough estimate.)

Large D limit

Dual theory: $S_{\text{AdS}} = S_{\text{grav.+}\phi} + S_{\text{X}} + S_{\text{gauge}} + S_{\text{“SM”}} + S_{\text{int. “SM” + X}}$

S_{X} is considered (without fluctuations); $S_{\text{grav.+}\phi}$ and S_{gauge} are left

$$S_{\text{grav.+}\phi} = \frac{1}{l_P^3}\int d^5x\sqrt{|g|}e^{2\phi}\Big[-R + 2|\Lambda| - 4g^{ab}\partial_a\phi\partial_b\phi - V_\phi(\phi)\Big]$$

$S_{\text{grav.+}\phi}$ within large D limit: $g_{ab} = g^{(0)}_{ab} + \frac{1}{D}g^{(1)}_{ab} + \frac{1}{D^2}g^{(2)}_{ab} + \dots$ [Emparan'20]

near-AdS$_d$ (Poincaré patch): $A(z) = 1 + o(e^D)$, $B(z) = 1 + o(e^D)$, $f(z) = 1 - \left(\frac{z}{z_H}\right)^{D-1} + \mathcal{O}(e^D)$

$$ds^2 = A(z)\left(-f(z)dt^2 + \frac{dz^2}{f(z)} + B(z)\,\mathrm{d}\Omega_{D-2}\right) \xrightarrow{D\to\infty} \frac{l^2}{D^2}\left(-y^2d\tau^2 + dy^2\right) + o(e^{-D})\mathrm{d}\Omega_{D-2}$$

The simplest case for dilaton in AdS $\phi'' - (\phi')^2 + m^2(\phi^2 + 2\phi) = 0$ no Lie symmetries

Large D limit for black branes and GHY

For the black brane with normal n^A, tangent velocity u^A and extrinsic curvature K_{AB}

$$\left(\frac{\nabla^2 u_A}{K} - \frac{\nabla_A K}{K} + u^A K_{AB} - u^B \nabla_B u^A\right)\mathcal{P}^A_C = \mathcal{O}\left(\frac{1}{D}\right), \quad \mathcal{P}^{AB} = \delta^{AB} + u^A u^B$$

for "flat" metric η_{AB} (i.e. without any horizon) [Bhattacharyya'16]

$$g_{AB} = \eta_{AB} + \frac{(n^A - u^A)(n^B - u^B)}{\psi^{D-3}} + \mathcal{O}\left(\frac{1}{D}\right), \quad \psi\big|_{\text{horizon}} = 1 \text{ is a scalar function}$$

$$S_{\text{grav.}} = \underbrace{\int_{\mathcal{M}} R\, d\text{Vol}_D}_{\substack{\text{EH without BH} \\ \text{no PT}}} + \underbrace{\int_{\partial\mathcal{M}} K\, d\text{Vol}_{D-1}}_{\substack{\text{GHY} \\ \text{with PT}}}$$

$\boxed{1}$ $\Delta F_{\text{grav}} \propto \Delta S_{\text{GHY}}$; $\boxed{2}$ $S_{\text{matter}}[g_{AB}]$; $\boxed{3}$ S.T. with $D8/\overline{D8}/D4$.

Effective CFT within large D limit

$$\underbrace{\int \frac{d^{d+1}x\sqrt{-g}}{16\pi G_{d+1}}\left(R+\frac{d(d-1)}{L^2}-\frac{L^2}{g_s^2}F^2\right)}_{\text{Einstein-Maxwell}\quad \text{RN-AdS}_{d+1}} \to \underbrace{\int \frac{d^2x\sqrt{-h}e^{-2\psi}}{16\pi G_2}\left(R_2-\frac{1}{g_s^2}F^2+4(\nabla\psi)^2+\frac{1}{\tilde{L}^2}\right)}_{\text{Einstein-Maxwell-dilaton}\quad \text{RN-AdS}_2}$$

$$T^{ab}\Big|_{\partial\text{RN-AdS}_2} = \frac{2}{\sqrt{-h}}\frac{\delta S\big|_{\partial\text{RN-AdS}_2}}{\delta h_{ab}};$$

asymptotic diffeomorphism (gauge invariant) $x^\mu \to x^\mu + \epsilon^\mu$

$$(\delta_\epsilon+\delta_\Lambda)T_{tt} = \text{usual transformation} + \text{anomalous term}, \qquad A_\mu \to A_\mu + \partial_\mu\Lambda$$

anomalous term $\Rightarrow$ $c = \frac{3}{2\pi G_2} = \frac{3\Omega_{d-1}}{2\pi G_{d+1}} r_0^{d-1}$ $\Rightarrow$ $s_{\text{CFT}} = \frac{\pi^3}{3}Tc = \frac{1}{4G_2}$ (by Cardy formula)

CFT$_{d=1}$ reproduces Bekenstein-Hawking for RN-AdS$_{d+1}$ $\quad s = \frac{\pi^3}{3}Tc = \frac{1}{4G_2} = \frac{\text{Area}}{G_{d+1}}$ [Guo'16]

Hawking-Page PT within large D limit eff. CFT

something that looks like a conclusion

$$F_{\text{thermal AdS}} = -pV \xrightarrow[\text{phase transition}]{\text{Hawking-Page}} F_{\text{BH}} = M - TS - \mu Q - pV$$

1. $S_{\phi\text{+grav}} + S_{\text{gauge}}$ asymptotic RN-AdS$_{d+1}$ (unknown) solutions (or “near-AdS$\times$any-compact” for top-down approach),
2. $\xrightarrow{\text{large D}}$ RN-AdS$_2$ (RN-AdS$_3$) solutions in leading $1/d$ order,
3. $\xrightarrow[\text{asymptotic}]{\text{border}}$ central charge of CFT$_1$ (CFT $_2$),
4. $\xrightarrow[\text{formula}]{\text{Cardy's}}$ free energy and Hawking-Page phase transition (in RN-AdS$_{d+1}$),
5. $\overset{\text{AdS/CFT}}{\rightsquigarrow}$ confinement/deconfinement-like PT for CH model (as it works for AdS/QCD).

Thank you for your attention!

Phys. Rev. D 108, 115011 (matter sector only)

System of units: $1 = c = \hbar = k_B = \pi = e = i = -1 = 2\text{kg}$

Holographic model

$\mathcal{L}_{\text{CH}}$ – strongly coupled $\quad \Rightarrow \quad$ consider $N \gg 1$ $\quad \Rightarrow \quad$ $\mathcal{Z}_{\text{CH}}[J] = \mathcal{Z}_{\text{AdS}}[J]$

The dual theory: $\mathcal{Z}_{\text{AdS}}[J] \sim \exp\Big(- S_{\text{AdS}}[J] \Big)$ is weakly coupled $\Rightarrow$ quasiclassical limit

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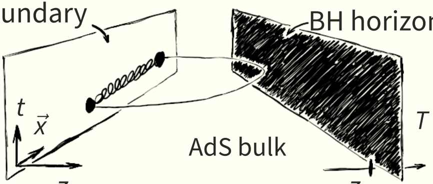
The asymptotic behavior near the conformal border ∂AdS of the dual theory fields **defines the sources** of the CH operators (i.e. the **correlator functions**)

$$X_{IJ} \overset{z\to 0}{\sim} \frac{\sqrt{N}}{2\pi} J_{IJ} z + \frac{2\pi}{\sqrt{N}} \Sigma_{IJ} z^3 + \dots \qquad X_{IJ} : \text{AdS}_5 \overset{\text{dual}}{\Longleftrightarrow} \Sigma_{IJ} : \mathbb{R}^{1,3}$$

Holography is the **duality** between **strongly coupled** theory on the border and **weakly coupled** (quasiclassical) bulk theory.

$F = -T \log \mathcal{Z}_{\text{CH}} \sim T S_{\text{AdS}} \propto \text{Vol}_4 \cdot \mathcal{F}$ In homogeneous case ($\chi = \chi(z)$): $\mathcal{F} \propto V_{\text{eff}}[\chi]$

Finite temperature



$$\mathcal{Z}[J] = \int \mathcal{D}[\ldots]e^{-S-\mathcal{O}\cdot J} \overset{\text{AdS/CFT}}{=\!=} \exp\left(-S_{\text{AdS}}[\psi]\big|_{z=0}\right), \quad J \overset{\text{AdS/CFT}}{=\!=} \psi_0(x), \quad \langle\mathcal{O}\rangle \overset{\text{AdS/CFT}}{=\!=} \psi_1(x)$$

EoM solutions $\xrightarrow[\text{asymptotic}]{\partial\text{AdS}}$ boundary part $S_{\partial\text{AdS}}$ $\xrightarrow[\text{approach}]{\text{quasiclassical}}$ CFT generating function

Action of the holographic model

$$S_{\rm tot} = S_{\rm grav+\phi} + S_{\rm X} + S_{\rm A} + S_{\rm SM} + S_{\rm int}, \quad S_{\rm A} = -\frac{1}{g_5^2}\int d^5x\sqrt{|g|}e^{\phi}g^{ac}g^{bd}F_{ab}F_{cd}$$

$$S_{\rm grav+\phi} = \frac{1}{l_P^3}\int d^5x\sqrt{|g|}e^{2\phi}\Big[-R + 2|\Lambda| - 4g^{ab}\partial_a\phi\partial_b\phi - V_\phi(\phi)\Big], \quad a,b = 0,\dots 4$$

$$S_{\rm int} = \epsilon^4\int_{z=\epsilon} d^4x\sqrt{|g^{(4)}|}\left[c_Y B_\mu \,\mathrm{Tr}\left(T_Y A^\mu\right) + c_W W_{k,\mu}\,\mathrm{Tr}\left(T_k A^\mu\right) + \mathcal{L}_\psi\right]$$

$$S_{\rm X} = \frac{1}{k_s}\int d^5x\sqrt{|g|}e^{\phi}\left[\frac{1}{2}g^{ab}\,\mathrm{Tr}\left(\nabla_a X^T\nabla_b X\right) - V_X(X)\right], \quad \nabla_a X = \partial_a X + [A_a, X], \quad A_a = 0$$

$$V_X(X) = \mathrm{Tr}\left(-\frac{3}{2L^2}X^TX - \frac{\alpha}{4}(X^TX)^2 + L^2\frac{\beta}{6}(X^TX)^3 + O(X^8)\right)$$

$$L\cdot X_{IJ} \sim \frac{\sqrt{N}}{2\pi}J_{IJ}\tilde{z} + \frac{2\pi}{\sqrt{N}}\Sigma_{IJ}\tilde{z}^3 + \dots$$

Geometry

$$S_{\text{grav}+\phi} = \frac{1}{l_P^3}\int d^5x\sqrt{|g|}e^{2\phi}\Big[-R+2|\Lambda|-4g^{ab}\partial_a\phi\partial_b\phi - V_\phi(\phi)\Big], \quad a,b=0,\dots 4$$

$$ds^2 = \frac{L^2}{\tilde{z}^2}A(\tilde{z})^2\left(f(\tilde{z})d\tau^2+\frac{d\tilde{z}^2}{f(\tilde{z})}+d\vec{x}^2\right), \quad \phi=\phi(\tilde{z})$$

$$f = 1-\frac{\tilde{z}^4}{z_H^4}, \quad \phi=\tilde{\phi}_2\tilde{z}^2, \quad z_H=\frac{1}{\pi T}.$$

Effective field theory

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S-J\cdot\phi} =: e^{W[J]}$$

$$\Gamma[\langle\phi\rangle] = W[J] - \frac{\delta W[J]}{\delta J}\cdot J = \int_X d^d x \Big(\underbrace{K_{\text{eff}}[\partial\langle\phi\rangle]}_{=0 \text{ if } \langle\phi\rangle = \text{const}} + V_{\text{eff}}[\langle\phi\rangle] \Big)$$ – effective action

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Effective potential: $V_{\text{eff}} = \frac{1}{\text{Vol}_4}\Gamma$

Equation of motion (EoM): $\frac{\delta\Gamma}{\delta\langle\phi\rangle} = J \overset{\langle\phi\rangle=\text{const}}{=\!=\!=} \frac{\delta V_{\text{eff}}}{\delta\langle\phi\rangle} \overset{J=0}{=\!=} 0$ gives extrema condition

Effective potential

Extrema condition: $V_{\text{eff}}\big|_{\text{extrema}} = V_{\text{eff}}\big|_{J=0} \Leftrightarrow G_0$;
AdS/CFT: $G_0 \Leftarrow$ boundary term of dual theory $S_{\partial\text{AdS}}$

$$\text{Vol}_X\, V_{\text{eff}}\big|_{\text{extrema}} = G_0 = W[J=0] \overset{\text{AdS/CFT}}{=\!=} S_{\text{AdS}}\Big|_{\partial\text{AdS}}^{\psi_0=0}$$

Extrema condition & duality: $J = \psi_0 = 0$; duality: $\langle\phi\rangle = \psi_1$

$$\frac{\delta V_{\text{eff}}}{\delta\langle\phi\rangle} \overset{\text{AdS/CFT}}{=\!=} \frac{\delta}{\delta\psi_1}\left(S[\psi]\big|_{\partial\text{AdS}}\right)\Big|_{\psi_0=0} = 0 \quad \left(\begin{array}{c}\text{with assumption}\\ \langle\phi\rangle=\text{const}\end{array}\right)$$

gives vacuum expectation values: $\left\{\langle\phi\rangle_{\text{min 1}}, \langle\phi\rangle_{\text{min 2}}, \dots\right\}$ — possible vacuums

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Extrema positions and values $\left\{\left(\langle\phi\rangle_{\text{min i}}, V_{\text{eff}}[\langle\phi\rangle_{\text{min i}}]\right)\right\} \Rightarrow$ phase transitions

Gravitational Waves

The spectrum of the gravitational waves can be estimated as
(within the approach of **relativistic** velocity of the bubble walls $v_w \sim 1$)

$$\Omega_{GW} h^2 = 1.67 \cdot 10^{-5} \kappa \Delta \left(\frac{\beta}{H_*}\right)^{-2} \left(\frac{\alpha}{1+\alpha}\right)^{2} \left(\frac{g_*}{100}\right)^{-\frac{1}{3}}$$

Only scalar waves! Sound waves and turbulence are not included!
We **estimate** only scalar waves produced during initial collisions.

$$f_0 = 1.65 \cdot 10^{-5} \text{Hz} \cdot \frac{f_*}{\beta} \frac{\beta}{H_*} \frac{T}{0.1\text{TeV}} \left(\frac{g_*}{100}\right)^{\frac{1}{6}} \text{ Hz}$$

$(\Omega_{GW} h^2, f_0)$-curve is the estimation GW amplitude (peak value).
It does not contain the spectral shape $S(f_0)$ (in this case $S(f_0 = f_0^{\text{peak}}) = 1$).

Temperature estimations

Experimental restrictions $\Leftrightarrow$ mass of the **lightest predicted** particle.

$$\Sigma_{IJ} = \xi^\top \left[\begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & \varsigma \end{pmatrix} + \eta_i \tilde{T}_i \right] \xi \underbrace{\overset{\text{AdS/CFT}}{}}_{\text{dual to}} X_{IJ} \to \begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & \chi \end{pmatrix}$$

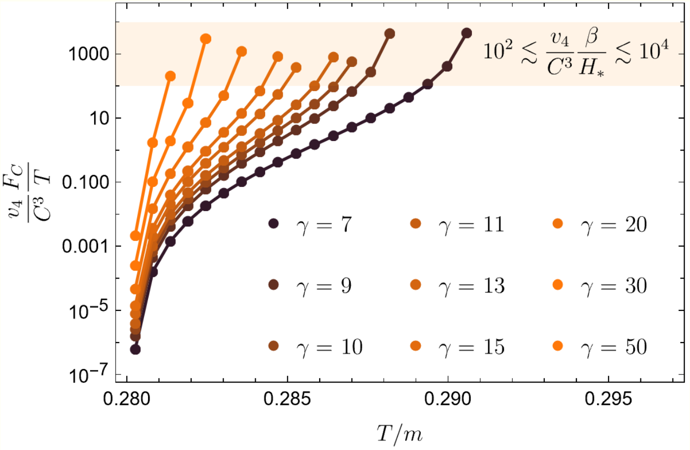
$m_\eta \sim m_{\delta\chi}$ fluctuation mass $\sim$ slope of the "hat".

$$\chi(z) \to \chi(z) + \delta\chi(t, \vec{x}, z) \quad \Rightarrow \quad \text{EoM}_z[\chi] \to \text{EoM}_{t,\vec{x},z}[\chi + \delta\chi] \xRightarrow{\partial\text{AdS}} \boxed{2m^2 = \phi_2}$$

$$T = \frac{1}{\pi}\frac{1}{z_\mathrm{H}} \quad \Rightarrow \quad T = \frac{m}{\pi}\sqrt{\frac{2}{\phi_2}}, \quad z_\mathrm{H}^2 = \frac{\phi_2}{2m^2}$$

Bubble free energy

Free energy of a bubble: $F[V_{\text{eff}}] \overset{\text{thin walls}}{\underset{\text{approximation}}{=\!=\!=}} 4\pi R^2 \mu - \frac{3\pi}{4} R^3 \left(\mathcal{F}_{\text{out}} - \mathcal{F}_{\text{in}}\right)$



$F_C \overset{\text{def}}{=} F(R_C)$: if $R > R_C$, bubbles grow and **PT occurs**.

$$\frac{\beta}{H*} \sim \frac{F_C}{T} + \mathcal{O}(T)$$

$1/\beta \sim$ appear $\to$ collide time
$1/H_* \sim$ universe expansion

$v_4 \sim 10^{-1} \ll 1, \quad C \sim 1$

$$\boxed{10^3 \gtrsim \frac{\beta}{H_*} \gtrsim 10^5}$$

Holographic effective potential

$$\mathcal{Z}_{\mathrm{CH}}[J] = \int \mathcal{D}\varphi \, \exp\left(-S[\varphi] - \int d^4x \, \varphi(x)\, J(x) \right) \overset{\text{def}}{=} e^{-W[J]}$$

$$\langle\varphi\rangle = \left.\frac{\delta W[J]}{\delta J}\right|_{J=0}, \quad \Gamma[\langle\varphi\rangle] = W[J] - \int d^4x \, \frac{\delta W[J]}{\delta J(x)} J(x)$$ – Effective Action

EoM: $\dfrac{\delta\Gamma}{\delta\langle\varphi\rangle} = J$

extrema condition

Homogeneus Solution $\Rightarrow \langle\varphi\rangle = \mathrm{const}_{\mathbb{R}^{1,3}} \Rightarrow \Gamma = -\mathrm{Vol}_4 V_{\mathrm{eff}}$ — Effective Potential

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$\mathcal{Z}[J]$ $\underset{\text{correspondence}}{\overset{\text{AdS/CFT}}{=\!=}}$ $\mathcal{Z}_{\mathrm{AdS}}$ $\underset{\text{approximation}}{\overset{\text{quasiclassical}}{\approx}}$ $e^{-S_{\mathrm{AdS}}}\big|_{\partial\mathrm{AdS}}$ — **quasiclassical non-perturbative**

$$V_{\mathrm{eff}} = -\frac{1}{\mathrm{Vol}_4} S_{\mathrm{AdS}}\Big|_{\partial\mathrm{AdS}}$$

— boundary term of the bulk theory defines *quantum* effective potential

“Extrema” curves

$$\frac{\delta S_\chi}{\delta \chi} = 0 \Rightarrow \chi \xrightarrow{z\to 0} Jz + \left(\sigma - \left(\frac{3}{2}J^3 + \phi_2 J\right)\log z\right) z^3 + o(z^5)$$ — give the sourses for CFT operators

Knowing the *extrema* of the effective potential and its *values* at these points, we can judge abut the phase transition

$$V_{\text{eff}} = -\frac{1}{\text{Vol}_4} S_\chi\Big|_{\partial\text{AdS}} \Rightarrow$$ from EoM for effective action : $\text{Vol}_4 \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = J \Rightarrow$ extrema condition is absence of sources $\Rightarrow J = 0$

$$\overbrace{\chi \xrightarrow{z\to 0} \sigma z^3 + o(z^5)}^{\text{“extreme” solutions}} \text{ must give } \overbrace{\frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = 0}^{\text{extrema}} \Rightarrow$$ a new condition for ϕ_2 and $\langle\varphi\rangle$

$$T \sim \frac{1}{\sqrt{\phi_2}}, \quad \frac{\delta V_{\text{eff}}}{\delta\langle\varphi\rangle} = 0 = \frac{\delta}{\delta\sigma} S_\chi\left[\chi_{\text{Sol.}}(z; J, \sigma)\right]\Big|_{J=0} \Rightarrow \{\sigma_1, \ldots, \sigma_n\}$$ – extrema

σ is (source) dual to $\langle\varphi\rangle$, vacuum average of the effective theory

Nucleation ratio

Baryogenesis generates enough asymmetry (enough efficient)
if there is one bubble per Hubble volume

$$\underbrace{\text{Nucleation Ratio}: \quad A T^4 e^{-\frac{F_C}{T}}}_{\substack{\text{Bubbles produced} \\ \text{per time} \times \text{space volume}}} \sim \underbrace{H^4(T) = \left(\frac{T^2}{M_{\text{Pl}}}\right)^4 \quad \text{– Expansion of the Univerce}}_{1/(\text{Hubble time} \times \text{volume})}$$

$F = F[\langle\varphi\rangle, R]$ – Free energy of the bubble; R is the radius of the bubble

Hubble horizon (time, volume, radius) — speed of receding object behind it is greater than the speed of light (Don't confuse with cosmological horizon)

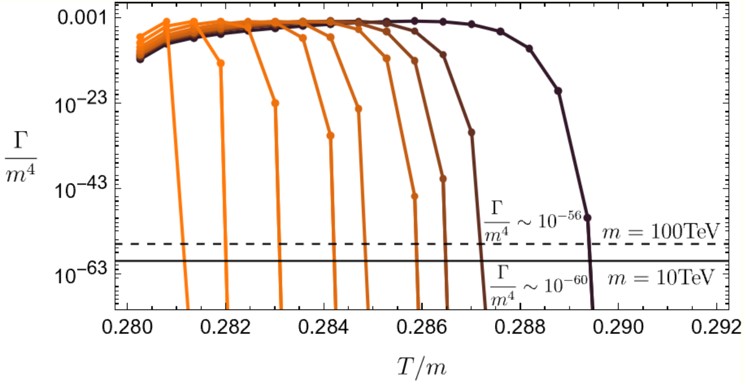
Bubble appears with a certain size. It defines with "micro-physics".
If its radius is grater, then critical one $\left.\frac{\partial F}{\partial R}\right|_{R_C \overset{\text{def}}{=} R}$, the bubble grow. Otherwise, it bursts.

It gives $F_C \overset{\text{def}}{=} F(R_C)$ and defines nucleation ratio and "viability of the model".

Estimations of the nucleation ratio

$\Gamma \sim H_*^4, \quad \frac{\Gamma}{m^4} \sim \frac{H_*^4}{m^4} \propto \frac{m^4}{M_{\rm Pl}};$

F_C is defined with an error, so $e^{\frac{F_C}{T}}$ has large error



Potentials of CFT and the dual theory

$$V_\chi = a_2\chi^2 + a_4\chi^4 + a_6\chi^6, \quad a_2 < 0,\ a_4 < 0,\ a_6 > 0 \quad \textbf{no barrier}$$

$$V_{\text{eff}} = b_2\langle\varphi\rangle^2 + b_4\langle\varphi\rangle^4 + b_6\langle\varphi\rangle^6, \quad b_2 > 0,\ b_4 < 0,\ b_6 > 0 \quad \textbf{there's a barrier}$$

in details:

- $V_{\text{eff}} = V_{\text{eff}}[\langle\varphi\rangle]$ describes a quantum objects at the border. V_χ is a dual classical potential in the bulk.
- $V_{\text{eff}} = -\frac{1}{\text{Vol}_4} S_{\text{AdS}}\Big|_{\partial\text{AdS}}$ includes the solutions of the EoM $\frac{\delta S_\chi}{\delta\chi=0}$ in bulk. In other words, V_{eff} includes physics of AdS

Conditions for the dual theory potential

$V_\chi(\chi) = \frac{m^2}{2}\chi^2 - \frac{D}{4L^2}\lambda\chi^4 + \frac{\lambda^2\gamma}{6L^2}\chi^6$ is the expantion of a more general theory

Suggestions:

- The potential V_χ always has true vacuum with $E_{\min}$ ($V_\chi \xrightarrow{\chi\to\pm\infty} \infty$). So we may use any even power χ^n instead of the last term χ^6.
- The expansion of V_χ has certain sign of the second term $\lambda > 0$ (the first one m^2 chosen for the theory to be conformal in AdS).
- Higher orders of the expansion don't give new minima at the considered temperatures.

The certain parametrization has been chosen with respect to the "symmetries"

"Scale invariace", defining the coeffcicents : $L \to L'$; $\chi \to \sqrt{\lambda}\chi$;

Conformality near the AdS border ("correct" conformal weights) : $\Delta_- = 1$, $\Delta_+ = 3$ $\Rightarrow m^2 = -\frac{D}{3L^2}$

D is for *the Large D limit*. But its usage doesn't give any results. (to keep interaction constants finite at $D \to \infty$)

SM - CH model interactions

$$F \underset{\text{approximation}}{\overset{\text{thin walls}}{=\!=}} 4\pi R^2\mu - \frac{3\pi}{4}R^3\left(\mathcal{F}_{\text{out}} - \mathcal{F}_{\text{in}}\right)$$ — **physical units are required**

- Fix the Parameters (Interaction with Standard Model – bulk gauge fields)
- Physical Units (Infrared Regularization and finite temperature – “radial” heavy fluctuations)

$$W^{\alpha}_{\mu} J^{\alpha\,\mu}_{L} + B_{\mu} J^{\mu}_{Y} \quad \Leftrightarrow \quad J^{\mu} \sim A^{M} \text{ – bulk } \mathcal{G} \text{ gauge field}$$

The physical values can be estimated without gauge field:

$$\Sigma_{IJ} = \langle \bar{\Psi}_I \Psi_J \rangle = \xi^{\top}\left[\begin{pmatrix} 0_{4\times 4} & 0 \\ 0 & X \end{pmatrix} + \eta_i \tilde{T}_i\right]\xi \quad \Leftrightarrow \quad \frac{1}{T} \propto \sqrt{\phi_2} \sim \mu_{\text{IR}} \sim m_{\eta} \gtrsim 10 \text{ TeV}$$

$$m_{\eta} \quad \Leftarrow \quad X \to X + \delta X \text{ – correction of the background field} \quad \Rightarrow \quad \eta \text{ – pNG boson}$$

CH gauge field

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{CH}} + \underbrace{B_\mu \operatorname{Tr}\left(T_Y \hat{J}^\mu\right) + W_{k,\mu} \operatorname{Tr}\left(T_k \hat{J}^\mu\right) + \sum_r \bar{\psi}_r \mathcal{O}_r + \text{h.c.}}_{=\mathcal{L}_{\text{interactions}}}$$

$$SO(5) \times U(1) : A_M = A_M^K T^K + A_{M,Y} T_Y$$

$$SO(5) \to SO(4) : \quad \underbrace{A_M^K T^K}_{\in SO(5)} \to \underbrace{A_M^a T^a}_{\in SO(4)} + \underbrace{A_M^i T^i}_{\in SO(5)/SO(4)}$$

$$SO(4) \cong SU(2) \times SU(2) : \quad A_M^K T^K = \underbrace{A_M^{k,L} T_L^k}_{\in SU(2)_L} + \underbrace{A_M^{k,R} T_R^k}_{\in SU(2)_R}$$

conserved currents: $\hat{J}_\mu \overset{\text{dual}}{\Longleftrightarrow} A_\mu(t,x,z)\Big|_{z=0}^{\partial\text{AdS}}$, holographic gauge: $A_z = 0$

$\mathcal{O} \overset{\text{dual}}{\Longleftrightarrow} \mathcal{J}(A, \Psi, \phi, \ldots)$ — composite operators of the CH fields